VARDHAMAN COLLEGE OF ENGINEERING

(Autonomous)

Shamshabad, Hyderabad – 501 218

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

VCE – R 15 Regulations

IV Semester

Course Code	:	**A3212**			
Course Title	:	**CONTROL SYSTEMS**			
Course Structure	:	Lectures	Tutorials	Practicals	Credits
		3	1	-	4

Syllabus

UNIT - I

BASICS IN CONTROL SYSTEM AND TRANSFER FUNCTION: Introduction of Control Systems, Various types of systems (Open Loop and closed loop) and their differences- Classification and Feed-Back Characteristics of control system- Effects of feedback. Mathematical models – Differential equations, Translational and Rotational mechanical systems. Transfer Function of DC Servo motor - AC Servo motor- Synchro transmitter and Receiver.

UNIT - II

REPRESENTATION OF TRANSFER FUNCTION AND CONTROL DESIGN TECHNIQUES: Block diagram representation of systems considering electrical systems as examples. Block diagram algebra – Representation by Signal flow graph - Reduction using Mason's gain formula. Compensation techniques – Lag, Lead, Lead-Lag Controllers design, PID Controllers.

UNIT - III

TIME RESPONSE ANALYSIS: Standard test signals - Time response of first order systems – Characteristic Equation of Feedback control systems, Transient response of second order systems - Time domain specifications – Steady state response - Steady state errors and error constants – Effects of proportional derivative, proportional integral systems.

STABILITY ANALYSIS: The concept of stability – Routh's stability criterion – qualitative stability and conditional stability – limitations of Routh's stability. The root locus concept - construction of root loci-effects of adding poles and zeros to G(s) H(s) on the root loci.

UNIT - IV

FREQUENCY RESPONSE ANALYSIS: Introduction, Frequency domain specifications-Bode diagrams-Determination of Frequency domain specifications and transfer function from the Bode Diagram-Phase margin and Gain margin Stability Analysis from Bode Plots.

STABILITY ANALYSIS IN FREQUENCY DOMAIN: Polar Plots-Nyquist Plots-Stability Analysis.

UNIT - V

STATE SPACE ANALYSIS: Concepts of state, state variables and state model, derivation of state models from block diagrams, Diagonalization- Solving the Time invariant state Equations- State Transition Matrix and it's Properties – Concepts of Controllability and Observability.

TEXT BOOKS:

1. I. J. Nagrath, M .Gopal (2011), Control Systems Engineering, 5th edition, New Age International (P) Limited, New Delhi, India.
2. Benjamin. C. Kuo (2003), Automatic Control Systems, 8th edition, John Wiley and Son's, USA.

REFERENCE BOOKS:

1. K. Ogata (2008), Modern Control Engineering, 4th edition, Prentice Hall of India Pvt. Ltd, New Delhi.
2. N. K. Sinha (2008), Control Systems, 3rd edition, New Age International Limited Publishers, New Delhi.

UNIT - I

BASIC DEFINITIONS :

System :
An arrangement or combination of different physical components that are connected or related together to form an entire unit to achieve a certain objective is called a system. A kite is an example of a physical system, because it is made up of paper and sticks. A classroom is an example of a physical system.

Control :
The meaning of control is to regulate, direct or command a system so that a desired objective is obtained.

Plant :
It is defined as the portion of a system which is to be controlled or regulated. It is also called as process.

Controller :
It is the element of the system itself, or may be external to the system. It controls the plant or the process.

Input :
The applied signal or excitation signal that is applied to a control system to get a specified output is called input.

Output :
The actual response that is obtained from a control system due to the application of the input is termed as output.

Disturbances :
The signal that has some adverse effect on the value of the output of a system is called disturbance. If a disturbance is produced within the system, it is termed as an internal disturbance; otherwise, it is known as an external disturbance.

Control Systems :
It is an amalgamation of different physical elements linked in such a maner so as to regulate, direct or command itself to obtain a certain objective. A control system must have

(1) Input
(2) Output
(3) Ways to achieve input and output objectives and
(4) Control action

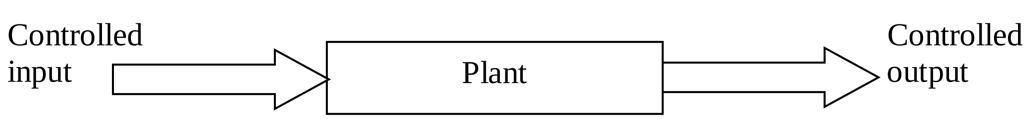


Fig. : Cause and effect relationship between the input and the output of a plant

Classification of Control Systems:

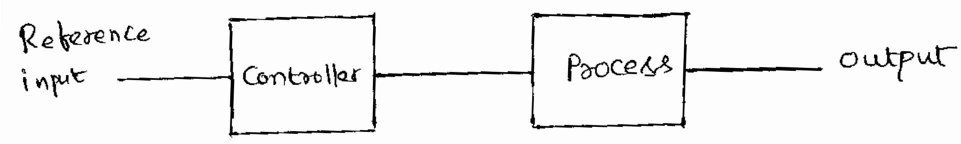
Based on controlling actions provided the control systems are classified as,

1. Open loop control system
2. Closed loop (or) Feedback control systems

   (a) Positive feedback (b) Negative feedback

OPEN-LOOP (OR) NON FEEDBACK (OR) MANUAL CONTROL SYSTEM:

The open loop control system can be described by a block diagram as shown in figure.



These are system in which the controlling actions are independent of output.
Examples: Traffic lights, washing machines and bread toaster etc (system having no sensors)

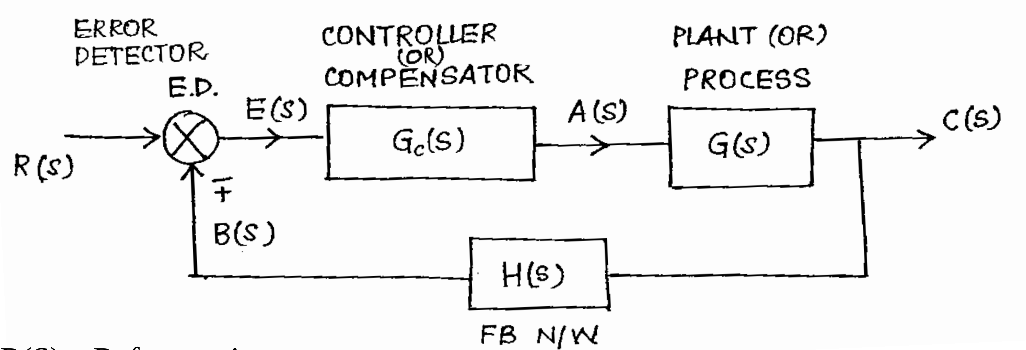
- **Advantages**
  · Simple in construction and design
  · Low cost
  · Easy to maintain
  · Usually not much troubled with the problems of instability
  · These systems are convenient to use when output is difficult to measure

- **Disadvantages:**
  · These are less accurate and their accuracy depends on the calibration
  · Inaccurate results are obtained with parameter variations within the system
  · Recalibration of the controller is required from time to time for maintaining accuracy

CLOSED-LOOP (OR) FEEDBACK (OR) AUTOMATIC CONTROL SYSTEMS:

It is a control system in which the controlling actions depends on the output.

Schematic representation of control system:



R(S) – Reference input
C(S) – Controlled output
E(S) – Error signal
B(S) – Feedback signal
A(S) – Actuating signal or moderating signal
G(S) – Plant Transfer function

$G_c(S)$ – Controller Transfer function
$G(S).G_c(S)$ – Forward path transfer function
$H(S)$ – Feedback path Transfer function
$G_c(S).G(S).H(S)$ – Loop (or) OLTF

$$\frac{C(S)}{R(S)} = \frac{G_C(S)\cdot G(S)}{1+G_C(S)\cdot G(S)}$$ Overall (or) CLTF

Examples: Temperature controllers, speed control of a motor etc. (system having sensors)

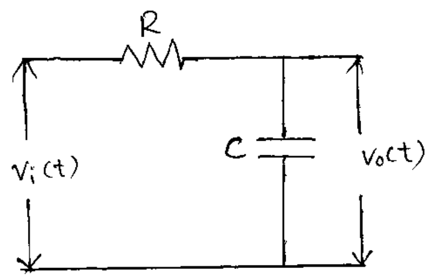
- **Advantages:**
  · Accuracy is more
  · Nonlinear distortions are less
  · Output is less sensitive to parameter changes with in the system
  · Bandwidth is increased.

- **Disadvantages:**
  · Complex in design
  · More expensive
  · May become unstable if there are malfunctions in the feedback

Non-feedback C.S.	Feedback C.S.
1. The changes in the output of the system cannot be corrected.	1. The changes in the output can be corrected.
2. No-sensor is available.	2. Sensor is available
3. NFCS are generally stable but cannot be stabilized is becomes unstable.	3. Due to disturbances, the system can become unstable but can be stabilized.
4. More sensitive for parameter variations.	4. Less sensitive for parameter variations
5. Simple & economical	5. Complex & costly

Mathematical models of physical system :

- Mathematical model which is the mathematical representation of the physical model through use of appropriate physical laws.
  Ex : RC network



$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)\,dt$ ……. (1)

$v_0(t) = \frac{1}{C}\int i(t)\,dt$ ……. (2)

} Mathematical model

Transfer Function :

The transfer function of a linear time invariant system is defined to be the ratio of the laplace transform of the output variable to the laplace transform of input variable under the assumption that all initial condition are zero.

Transfer Function = $\left.\dfrac{LT(output)}{LT(input)}\right|_{Initial\,conditions=0}$

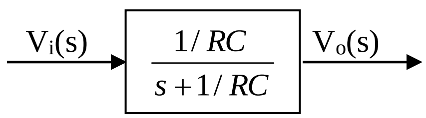
Taking the lapalce transform of above equations (1) and (2) (assuming zero initial conditions) we obtain

$V_i(s) = RI(s) + \dfrac{1}{Cs}\cdot I(s)$ …… (3)

$V_0(s) = \dfrac{1}{Cs}\cdot I(s)$ …… (4)

Then the transfer function is $\dfrac{V_0(s)}{V_i(s)} = G(s) = \dfrac{1}{RCs+1}$

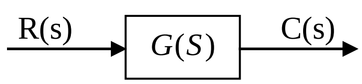
$$\therefore\ G(s) = \frac{1/RC}{s+1/RC}$$



Note :

- Transfer function of a system doesn't depend on the inputs to the system.

Impulse response and transfer function :



C(s) = G(s).R(s) ;

R(s) = L[δ (t)] = 1

C(s) = G(s)

C(t) = L^{-1} [G(s)]

C(t) = g(t)

where g(t) = Impulse response.

L[g(t)] = G(s)

- Linear Time invariant, the transfer function is the laplace transform of the impulse response assuming all the initial condition are zero.

Pole : It is a root of that denominator polynomial defined as negative of reciprocal of the system time constant which makes the transfer function infinity and represented as "X" in the s-plane.

Zero : It is a root of that numerator polynomial defined as negative of reciprocal of the system time constant which makes the transfer function zero and represented as "O" in the s-plane.

Type : The number of poles at the origin in an Open Loop transfer function is called as Type of the system.

Order : The highest power in the denominator polynomial or characterstic equation [1+G(S).H(S)] of Closed Loop control system is defined as the Order

(or)

The number of closed loop poles will indicates the Order of the system.

Procedure for deriving transfer functions :

1. It is assumed that there is no loading, i.e., no power is drawn at the output of the system.
2. The system should be approximated by a linear lumped constant parameters model by making suitable assumptions.

PROPERTIES OF TRANSFER FUNCTION :

- The ratio of the Laplace transform of output to input with all initial conditions to be zero is known as transfer function of system.
- The transfer function of a system is the Laplace transform of its imulse response under assumption of zero initial conditions.
- Replacing 's' variable with linear operation $D \equiv d/dt$ in transfer function of a system, the differential equation of the system can be obtained.
- The system poles and zeros can be determined from its transfer function.
- Stability can be found from characteristic equation.
- Transfer function cannot be defined for non-linear systems. It can be defined for linear systems only.

ADVANTAGES AND DISADVANTAGES OF TRANSFER FUNCTION :

Advantages

- Transfer function is mathematical model and it gives the gain of the system.
- Since Laplace transform is used, the terms are simple algebraic expressions and differential terms are not present.
- If transfer function of a system is known, the response of the system to any input can be determined very easily.
- Poles and zeros of a system can be determined from the knowledge of the transfer function of the system. Both poles and zeros have a vital role in the system's response.
- Transfer function helps in the study of stability analysis of the system.

Disadvantages

- Transfer function can be defined for linear systems only.
- Transfer function only defined under zero initial conditions.
- Transfer function describes the input, output behaviour of the system and doesn't give any information concerning the internal structure of the system.

The two important topics in the study of control systems are

1. Control system analysis

2. Control system design

By control system analysis we mean the investigation under specified conditions of the performance of the system.

By control system design we mean to find out one which accomplishes given task. If the performance is unsatisfactory it can be improved with the help of design. Whether it is control or not.

A control system is a physical system as it is a collection of physical objects connected through to serve an objective. The system can be electrical, mechanical or electromechanical. Examples of physical system can be cited from laboratory, industrial plant- an electronic amplifier composed of many components, the governing mechanism of a steam turbine or communication satellite orbiting the earth are all examples physical systems.

No physical system can be represented in its full physical intricacies and therefore idealizing assumption are always made for the purpose of analysis and synthesis of systems. An idealized physical system is called physical model. A physical system can be modeled in a number of ways depending up on specific problem to be dealt with and desired accuracy. For example an electronic amplifier may be modelled as an interconnection of linear lumped elements or some of these may be pictured as nonlinear elements in case the stress is on the study of distortion.

Once a physical model of a physical system is obtained, the next step is to obtain a mathematical model which is the mathematical representation of the physical model through the use of appropriate physical laws (Ohm's law, kirchoff's law, Newton's Law, Hooke's Law etc). Depending upon the choice of variables and the coordinate system, a given physical model may lead to different mathematical models. An electrical network, for example, may be modelled as a set of nodal equations using kirchoff's current law or a set of mesh equations using using kirchoff's voltage law. A control system may be modelled as a scalar differential equation.The particular mathematical model which gives a greater insight into the dynamic behaviour of physical system is selected.

When the mathematical model of a physical system is solved for given input, the result represents the dynamic response of the system.

A system is called linear if the principle of superposition applies the principle of superposition states that the response (output) produced by the simultaneous application of two different inputs is the sum of two individual responses (outputs). Hence for the $ linear system the response to several inputs can be calculated by treating one input at a time and adding the results A differential equation is linear if the co-efficient are constants or functions only of independent variable. If the coefficients of the describing differential equations are constants, the model is linear time- invariant.

$$6\frac{d^2x}{dt^2}+3\frac{dX}{dt}d2x+X=F$$

On the other hand if the coefficients of the coefficients of the describing differential equations are functions of time 't' (the independent variable) then the mathematical model is linear time - variant. An example is a missile. The mass of a missile changes due to fuel consumption.

$$t^2\frac{d^2x}{dt^2}+\frac{dX}{dt}+X=F$$

The transfer function of a linear time- invariant system is defined as the ratio of the laplace transform of the output (response) to the laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Consider the linear time invariant system defined by the following differential equation.

$$a_0\frac{d^nc}{dt^n}+a_1\frac{d^{n-1}c}{dt^{n-1}}+\ldots\ldots+a_{n-1}\frac{dc}{dt}+a_nC=b_0\frac{d^mr}{dt^m}+b_1\frac{d^{m-1}r}{dt^{m-1}}+b_{m-1}\frac{dr}{dt}+b_mr$$

Taking Laplace transform on both sides and assuming zero initial conditions,

$$\frac{C(s)}{R(s)}=\frac{b_0s^m+b_1s^{m-1}+\ldots\ldots+b_m}{a_0s^n+a_1s^{n-1}+\ldots\ldots+b_n}$$

Comments on transfer function

1. The transfer function is an expression relating the output and input of a linear time invariant system in terms of the system parameters and is a property of the system itself independent of the input.

2. It does not provide any information concerning the physical structure of the system (the transfer functions of many different physical systems can be identical).

3. The highest power of □ in the denominator of the transfer function is equal to the order of the system.

4. The transfer function between an input and output of a system is defined as the laplace transform of impulse.

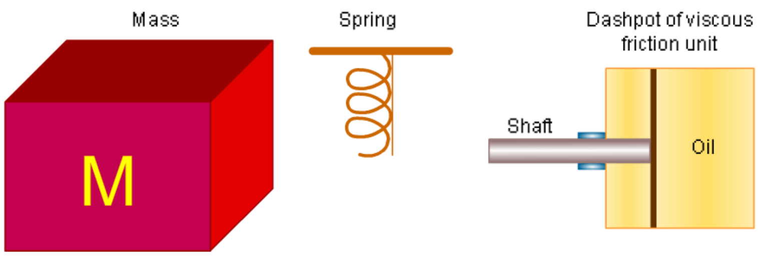
Modelling of the system:

The term mechanical translation is used to describe motion with a single degree of freedom or motion in a straight line. The basis for all translational motion analysis is Newton's second law of motion which states that the Netforce F acting on a body is related to its mass M and acceleration 'a' by the equation

$$\square\ F = Ma$$

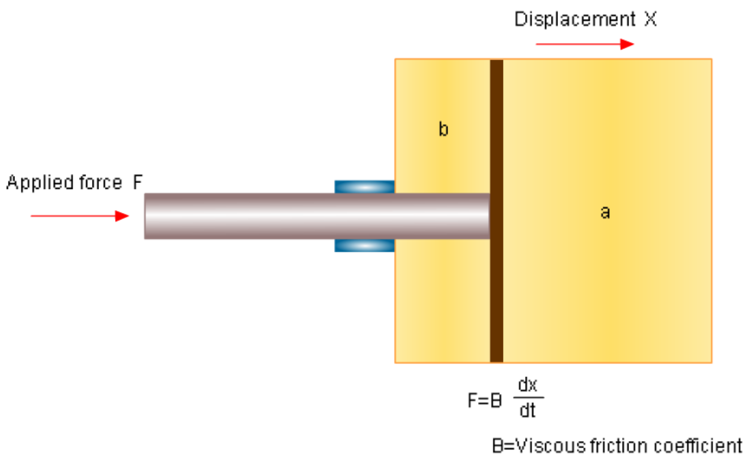
'Ma' is called reactive force and it acts in a direction opposite to that of acceleration. The summation of the forces must of course be algebraic and thus considerable care must be taken in writing the equation so that proper signs prefix the forces.

The three basic elements used in linear mechanical translational systems are (i) Masses (ii) springs (iii) dashpot or viscous friction units. The graphical and symbolic notations for all three are shown in the below figure.



The spring provides a restoring a force when a force F is applied to deform a coiled spring a reaction force is produced, which to bring it back to its freelength. As long as deformation is small, the spring behaves as a linear element. The reaction force is equal to the product of the stiffness k and the amount of deformation.

Whenever there is motion or tendency of motion between two elements, frictional forces exist. The frictional forces encountered in physical systems are usually of nonlinear nature. The characteristics of the frictional forces between two contacting surfaces often depend on the composition of the surfaces. The pressure between surfaces, their relative velocity and others. The friction encountered in physical systems may be of many types (Coulomb friction, static friction, viscous friction) but in control problems viscous friction, predominates. Viscous friction represents a retarding force i.e. it acts in a direction opposite to the velocity and it is linear relationship between applied force and velocity. The mathematical expression of viscous friction F=BV where B is viscous frictional co-efficient. It should be realized that friction is not always undesirable in physical systems. Sometimes it may be necessary to introduce friction intentionally to improve dynamic response of the system. Friction may be introduced intentionally in a system by use of dashpot as shown in the below figure. In automobiles shock absorber is nothing but dashpot.



The basic operation of a dashpot, in which the housing is filled with oil. If a force f is applied to the shaft, the piston presses against oil increasing the pressure on side 'b' and decreasing pressure side 'a' As a result the oil flows from side 'b' to side 'a' through the wall clearance. The friction coefficient B depends on the dimensions and the type of oil used.

For writing differential equations

- Assume that the system originally is in equilibrium in this way the often-troublesome effect of gravity is eliminated.

- Assume then that the system is given some arbitrary displacement if no distributing force is present.

- Draw a freebody diagram of the forces exerted on each mass in the system. There should be a separate diagram for each mass.

- Apply Newton's law of motion to each diagram using the convention that any force acting in the direction of the assumed displacement is positive is positive.

- Rearrange the equation in suitable form to solve by any convenient mathematical means.

The rotational motion of a body may be defined as motion about a fixed axis. The variables generally used to describe the motion of rotation are torque, angular displacement θ, angular velocity ω and angular acceleration α.

The three basic rotational mechanical components are

1. Moment of inertia J.
2. Torsional spring K.
3. Viscous friction f.

Moment of inertia J is considered as an indication of the property of an element, which stores the kinetic energy of rotational motion. The moment of inertia of a given element depends on geometric composition about the axis of rotation and its density. When a body is rotating a reactive torque is produced which is equal to the product of its moment of inertia (J) and angular acceleration and is given by $T = J\alpha$

A well known example of a torsional spring is a shaft which gets twisted when a torque is applied to it. $T_s = K\theta$, θ is angle of twist and K is torsional stiffness.

There is viscous friction whenever a body rotates in viscous contact with another body. This torque acts in opposite direction so that angular velocity is given by

$T = f\omega =$ Where ω = relative angular velocity between two bodies.
f = co efficient of viscous friction.

Consider the mechanical system and the electrical system shown in the below table.

The differential equation for mechanical system is

$$M + \frac{d^2x}{dt^2} + B\frac{dx}{dt} + KX = f(t) \qquad (1)$$

The differential equation for electrical system is

$$L + \frac{d^2q}{dt^2} + R\frac{d^2q}{dt^2} + \frac{q}{c} = e \qquad (2)$$

The analogy is here is called force voltage analogy

Table for conversion for force voltage analogy

Mechanical System	→	Electrical System
Force (torque)	→	Voltage
Mass (Moment of inertia)	→	Inductance
Viscous friction coefficient	→	Resistance
Spring constant	→	Capacitance
Displacement	→	Charge
Velocity	→	Current

Another useful analogy between electrical systems and mechanical systems is based on force - current analogy. Consider electrical and mechanical systems shown in the below table.

For mechanical system the differential equation is given by

$$M + \frac{d^2x}{dt^2} + B\frac{dx}{dt} + KX = f(t) \qquad (1)$$

For electrical system

$$C + \frac{d2x}{dt2} + \frac{1}{R} + \frac{d\Phi}{dt^2} + \frac{\Phi}{L} = I(t) \qquad (2)$$

Comparing equations (1) and (2) we find that the two systems are analogous systems. The analogy here is called force – current analogy. The analogous quantities are listed.

Table of conversion for force - current analogy

Mechanical System		Electrical System
Force (torque)	→	Current
Mass (Moment of inertia)	→	Capacitance
Viscous friction coefficient	→	Conductance
Spring constant	→	Inductance
Displacement (angular)	→	Flux
Velocity (angular)	→	Voltage

Although it is equally easy to write the equations for either form of system and thus equations for either form of system and thus there is no need to consider analogs, to simplify the analysis, there are significant advantages to the use of electrical analogues mechanical systems. For example it is not particularly convenient to setup a mechanical spring mass dashpot system and test its response in the laboratory because such components are not available in a wide variety of sizes, and are inconvenient to work with in any event. Since electrical components, as are current and voltage signals in a variety of forms for test inputs and since currents and voltages are accurately measured with ease, it is often convenient to study the response equivalent to the mechanical system of interest, adjusting component values as required to provide the desired results.

Important Points:

- The control element manipulates the actuating signal preferably to different power stages so as to drive or feed to the controlled system.
- Control elements plays a vital role to get the desired output.
- For low speed and high torque applications, hydraulic actuators are used.
- Electric actuators have inherent flexibility in electrical power transmission and have linear speed torque characteristics which is desired.
- Higher torque to inertia ratio indicates better dynamic response of the motor.
- In AC servomotors, if symmetrical components are used then the starting torque is proportional to E (rms value of the sinusoidal voltage).
- The time constants of the field-controlled dc motor is large than that of armature controlled because of high inductance of field winding.
- An example of electromagnetic transducer that converts angular position of a shaft into electric signal is a synchros. It is also known as selsyn or autosyn.

- Sensitivity of OLCS is morethan CLCS hence OLCS is more sensitive to parameter variations.
- In CLCS the sensitivity of feedback is more than the forward path sensitivity. So overall sensitivity reduces.

PRACTICE QUESTIONS:

1. The Impulse Response of a control system is $10e^{-3t}$ u(t) the transfer function is equal to

   A. $\frac{10}{s+3}$ B. $\frac{10}{s-3}$ C. $\frac{3}{s+10}$ D. $\frac{3}{s-10}$

2. If TF of a certain system is $\frac{1}{s^2+3s+2}$ then IR is

   A. $(e^{-t}+e^{-2t})$ u (t) B. $(e^{-t}-e^{-2t})$ u(t) C. 2 $(e^{-t}+e^{-2t})$ u(t) D. 2 $(e^{-t}-e^{-2t})$ u(t)

3. If Input is δ (t) and output is $10e^{-2t}$ u(t) then TF of the system is

   A. $\frac{10}{s+2}$ B. $\frac{2}{s+10}$ C. $\frac{1}{s+2}$ D. $\frac{10}{s-2}$

4. The impulse response of an initially relaxed linear system is e^{-2t} u(t). To produce a response of te^{-2t} u(t). The input must be equal to

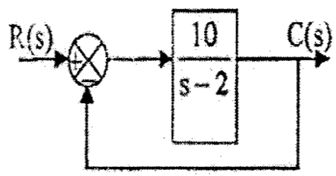
A. $2e^{-t}$ u (t) B. $\frac{1}{2}e^{-2t}$ u (t) C. e^{-2t} u (t) D. e^{-t} u (t) :

5. A linear time invariant system has an impulse response e^{2t}, t>0. If the initial conditions are zero and the Input is e^{3t}, then output for t > 0 is

A. $e^{3t}-e^{2t}$ B. e^{5t} C. $e^{3t}+e^{2t}$ D. none

6. The relationship between input x(t) and output y(t) of a system is given as $\frac{d^2y}{dt^2} = x(t-2) + \frac{d^2x}{dt^2}$ then TF of this system is A. $1+\frac{e^{-2s}}{s^2}$ B. $1+\frac{e^{2s}}{s^2}$ C. $1+s^2e^{-2s}$ D. $1+s^2e^{2s}$

7. Find the TF of the following diagram

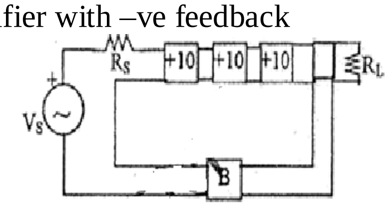


8 . If the CLTF of a unity feedback system is $\frac{4}{s^2+7s+13}$ then find the corresponding OLTF is…..

9. The unit-impulse response of a unity feedback control system is given by c (t) = $-te^{-t} + 2e^{-t}$ (t≥0) the open Loop transfer function is equal to

A. $\frac{s+1}{(s+2)^2}$ B. $\frac{2s+1}{s^2}$ C. $\frac{s+1}{(s+1)^2}$ D. $\frac{s+1}{s^2}$

10. Consider the following amplifier with –ve feedback

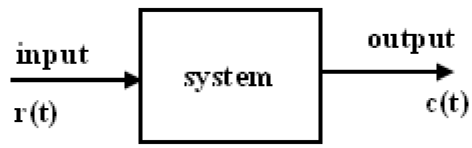


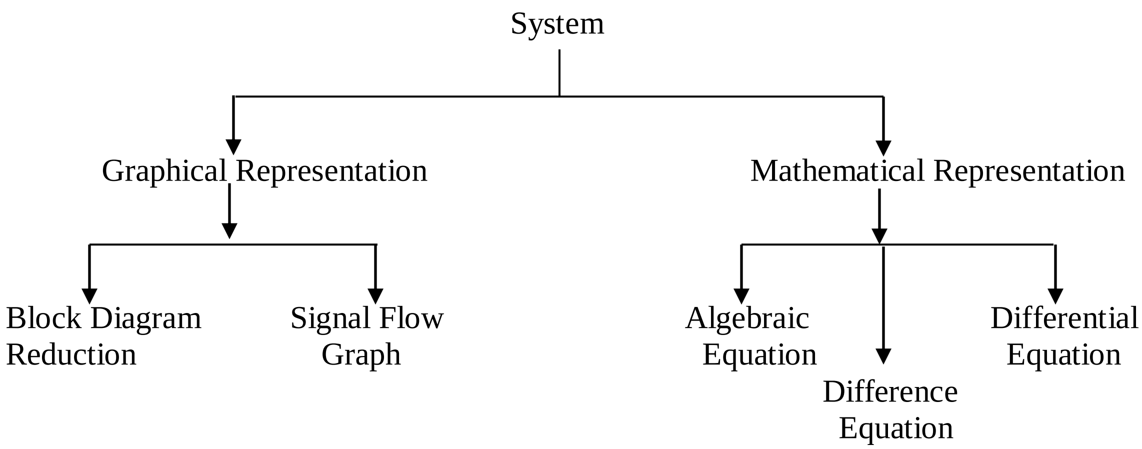
If the closed loop gain of the above amplifier is +100 the value of B will be

A. -9×10^{-3} B. $+9\times10^{-3}$ C. -11×10^{-3} D. $+11\times10^{-3}$

UNIT - II

SIGNAL FLOW GRAPH(SFG) AND BLOCK DIAGRAM(BD)

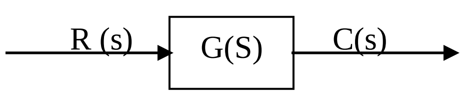




BLOCK DIAGRAM (BD):

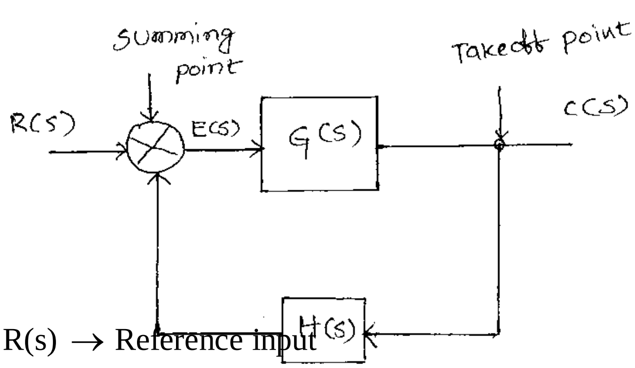
- The Block Diagram is a shorthand pictorial representation of the system between input and output.
- The purpose of Block Diagram is to find the overall transfer function of the system.

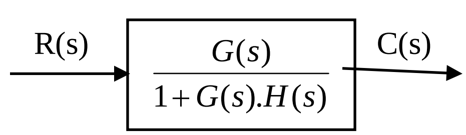
BLOCK DIAGRAM OF OPEN LOOP SYSTEM:



$$\frac{C(s)}{R(s)} = G(s)$$

BLOCK DIAGRAM OF CLOSED LOOP SYSTEM:





R(s) → Reference input

C(s) → Output signal (or) controlled variable
B(s) → Feed back signal
E(s) → Activating sign
G(s) → C(s) / E(s) = forward path transfer function
H(s) → Transfer function of feed back elements
G(s).H(s) → B(s)/E(s) → Loop transfer function
T(s) → C(s)/R(s) → Closed loop transfer function

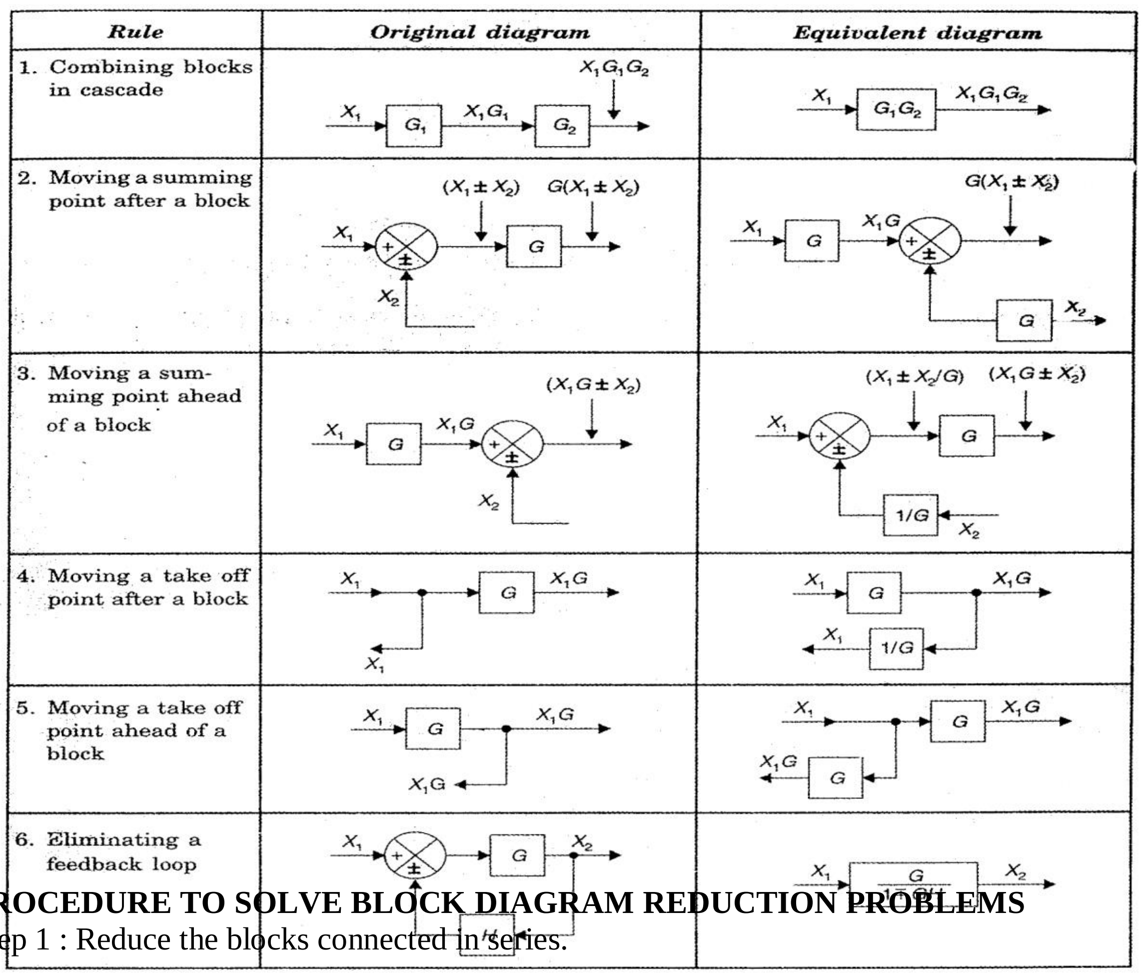
Negative Feed back :
The phase shift between input and feed back signal is (out of phase) $\pm\ 180^0$

- Accuracy in tracking steady state value.
- Rejection of disturbance signal
- Low sensitivity to parameter variation
- Reduction in gain

Positive Feed back :
The phase shift between the input and feed back signal is 0^0 (or) 360^0

- It increases the error signal and drives the output to instability
- Some time positive feedback is used in minor loops in control systems to amplify certain internal singals
- The positive feedback stable systems are multivibrators.

RUPES FOR BLOCK DIAGRAM REDUCTION



PROCEDURE TO SOLVE BLOCK DIAGRAM REDUCTION PROBLEMS

Step 1 : Reduce the blocks connected in series.

Step 2 : Reduce the blocks connected in parallel.
Step 3 : Reduce the minor internal feedback loops.
Step 4 : As far as possible try to shift take off point towards right and summing points to the left.
Step 5 : Repeat steps 1 to 4 till simple form is obtained, to get the final transfer function.
Step 6 : By using standard transfer function of simple closed loop system, obtain the closed loop transfer function $\frac{C(s)}{R(s)}$ of the overall system.

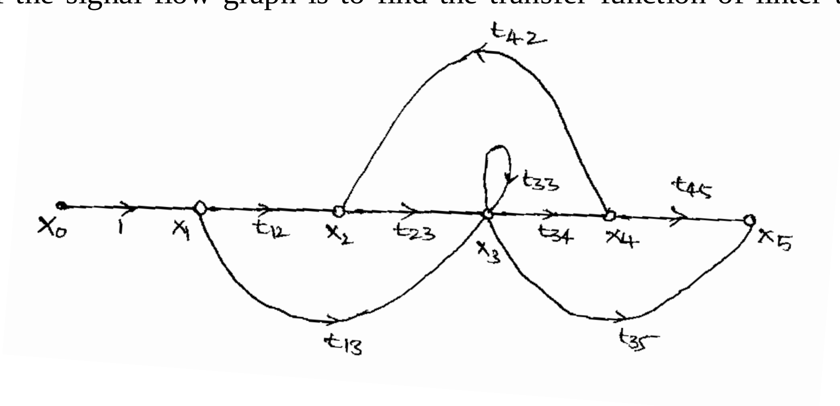
MULTI INPUT MULTI OUTPUT SYSTEMS :

Step 1 : Here reduce all but one inputs to zero. Find the resultant output
Step 2 : Repeat step 1 until all inputs are covered.
Step 3 : Find the resultant output by superposition.

SIGNAL FLOW GRAPH

Definition : Signal flow graph is a graphical technique that deals with the relation between variables of a system described in the form of set of linear algebraic equation.

- Signal Flow Graph is a graphical representation of the set of linear algebraic equations between input and output.
- Signal Flow Graph for representing the cause and effect of linear systems which are modified by algebraic equations.
- The purpose of the signal flow graph is to find the transfer function of linter time invariant systems.



Source Node :
The node having only outgoing branches is known as source or input node.
eg. x_0 is source node.

Sink Node :
The node having only incoming branches is known as sink or output node.
eg. x_5 is sink node

Chain Node :
A node having incoming and outging branches is known as chain node.
eg. x_1, x_2, x_3 and x_4

Forward Path :
A path from the input to output node is defined as forward path.
eg. $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5$ $\rightarrow$ 1st forward path

$x_0 \to x_1 \to x_3 \to x_4 \to x_5$ → 2nd forward path
$x_0 \to x_1 \to x_3 \to x_5$ → 3rd forward path
$x_0 \to x_1 \to x_2 \to x_3 \to x_5$ → 4th forward path

No node is to be traced twice

Feedback Loop :

A loop which originates and terminates at the same node is known as feedback path i.e. $x_2 \to x_3 \to x_4 \to x_2$. No node is to be trace twice

Self Loop :

A feedback loop consisting of only one node is called self loop.
i.e. t_{33} at x_3 is self loop.

Note :

A self loop cannot appear while defining a forward path or feedback path as node containing it gets traced twice which is not allowed.

Path gain :

The product of branch gains while going through a forward path is known as path gian i.e. path gain for path $x_0 \to x_1 \to x_2 \to x_3 \to x_4 \to x_5$ is $1.t_{12}..t_{23}.\ t_{34}.t_{45}$
This can also be called forward path gain.

Dummy Node :

If there exists incoming and outgoing branches both at 1st and last node, representing input and output variables, then as per definition these cannot be called as source or sink nodes. In such a case separate input and output nodes can be created by adding branches with gain 1. Such nodes are called as Dummy nodes.

Non-touching Loops :

If there is no node common in between the two or more loops, such loops are said to be non-touching loops

Loop Gain :

The product of all the gains of the branches forming a loop is called as loop gain. For a self loop, gain indicated along it is its gain. Generally, such loop gains are denoted by 'L'

MASON'S FORMULA :

$$TF = \sum_{K=1}^{n} \frac{P_K \cdot \Delta_K}{\Delta}$$

K = 1, 2, _ _ _ n
n = number of forward paths
P_K = Gain of K^{th} forward path

$$\Delta_K = 1 - \begin{bmatrix} Sum\ of\ all\ individual\ "loop"\ gains\ not \\ touching\ to\ k^{th}\ forward\ path \end{bmatrix}$$

$$+ \begin{bmatrix} SOP\ of\ "2-non\ touching\ loop"\ gains\ that \\ are\ not\ touching\ to\ k^{th}\ forward \\ path \end{bmatrix}$$

Δ = (q(s)) = Determinant of the graph

$$= 1 - \begin{bmatrix} \text{sum of all} \\ \text{individual} \\ \text{loop gains} \end{bmatrix} + \begin{bmatrix} \text{SOP of 2} \\ \text{Non-touching} \\ \text{loop gains} \end{bmatrix}$$

BLACK DIAGRAM TO SIGNAL FLOW GRAPH :

1. Name all the summing points and take off points in the block diagram.
2. Represent each summing and take off point by a separate node in signal flow graph.
3. Connect them by the branches instead of blocks, indicating block transfer functions as the gain of the corresponding branches.
4. Show the input and output nodes separately if required to complete signal flow graph.

DIFFERENCE BETWEEN BLOCK DIAGRAM AND SIGNAL FLOW GRAPH :

Sr. No.	Block Diagram	Signal Flow Graph
1.	Basic importance given is to the elements and their transfer functions.	Basic importance given is to the variables of the systems.
2.	Each element is represented by a block	Each variable is represented by a separate node
3.	Transfer function of the element is shown inside the corresponding block	The transfer function is shown along the branches connecting the nodes.
4.	Summing points and takeoff points are separate.	Summing and takeoff points are absent. Any node can have any number of incoming and outgoing branches.
5.	Feedback path is present from output to input.	Instead of feedback path, various feedback loops are considered for the analysis.
6.	For a minor feed back loop present, the formula $\frac{G}{1 \pm GH}$ can be used.	Gains of various forward paths and feedback loops are just the product of associative branch gains. No such formula $\frac{G}{1 \pm GH}$ is necessary.
7.	Block diagram reduction rules can be used to obtain the resultant transfser function.	The Mason's gain formula is available which can be used directly to get resultant transfer function without reduction of signal flow graph.
8.	Method is slightly complicated and time consuming as block diagram is required to be drawn time to time after each step of reduction.	No need to draw the signal flow graph again and again. Once drawn, use of Mason's gain formula gives the resultant transfer function.
9.	Concept of self loop is not existing in block diagram approach.	Self loops can exist in signal flow graph approach.
10.	Applicable only to linear time invariant systems.	Applicable to linear time invariant systems.

SENSITIVITY :

→ Sensitivity is the quantitative measure of amount of change in overall performance of a system for change in its parameters.

→ Denoted by letter 'S'

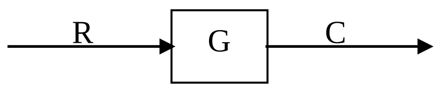
$$S_{()\ with\ respect\ to\ what}^{()\ of\ what} \qquad S_K^T = \frac{\left(\frac{\delta T}{T}\right)}{\left(\frac{\delta G}{G}\right)}$$

$$\boxed{S_G^T = \frac{\delta T}{\delta G}\cdot\frac{G}{T}}$$

G = System gain

T = Closed Loop Transfer Function

Open Loop system :



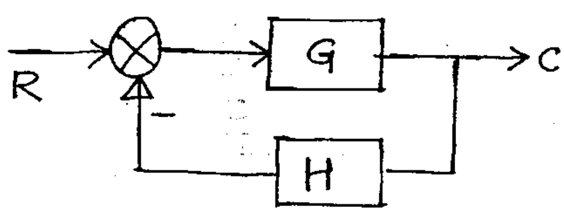
T = G

$$S_G^T = \frac{\left(\frac{\delta T}{T}\right)}{\left(\frac{\delta G}{G}\right)} = \frac{\delta T}{\delta G}\cdot\frac{G}{T} = 1.$$

S_G^T denotes the sensitivity of 'T' with respect to G

Open Loop Systems are highly sensitive.

Closed Loop system :



$$T = \frac{G}{1+GH}$$

The sensitivity with respect to G = $S_G^T = \frac{\delta T}{\delta G}\cdot\frac{G}{T} = \boxed{S_G^T = \frac{1}{1+GH}}$

Thus, the sensitivity of a closed loop system with respect to variation in G is reduced by a factor (1 + GH) as compared to open loop system.

Sensitivity with respect to H = $S_H^T = \frac{\delta T}{\delta H}\cdot\frac{H}{T} = \frac{-GH}{1+GH}$

- The above equation shows that for large value of GH sensitivity of feed back system with respect to H approaches unity. Thus the changes in H directly affect the system output.
- Sensitivity of feedback is more than the system. So that the sensitivity of the complete system becomes less.

Effect of Feedback :

1. Less effect of parameter variation on output
2. Response decays much faster which mans speed of the system response is fater.

3. Bandwidth increases
4. Effect of disturbance decreases

COMPENSATORS:

Purpose:

- If the system is unstable then required a compensator (or) controller to make it stable & achieve desired performance.
- If the system is stable then also required a compensator or controller to achieve the required or desired performance.

Compensators:

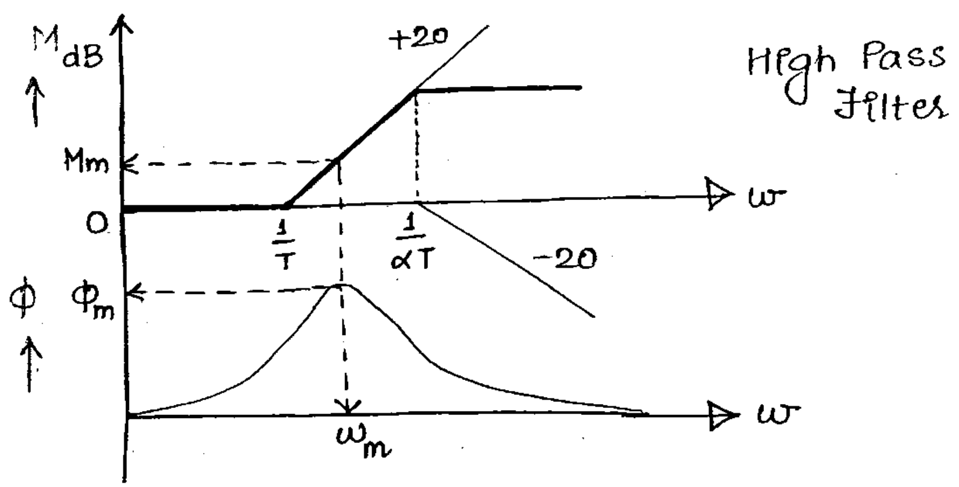
A compensator s a electrical network which adds the finite poles & finite zeros. So that the system performance is changed according to the requirement.

There are 3 types of compensator

1. Lead compensator
2. Lag compensator
3. Lag – Lead compensator

Lead Compensator:

- When a sinusoidal input is applied to a network it produces the sinusoidal steady state output having a phase lead with respect to input then it is called Lead Compensator.
- The Lead Compensator improves the transient behaviour of the system and improves the system stability.
- The disadvantage of lead compensator is it creates the attenuation in the system. To eliminate the attenuation we require to add a amplifier with the gain of $\frac{1}{\alpha}$.
- The frequency at which maximum phase lead occurs $\omega_m = \frac{1}{T\sqrt{\alpha}}$ rad/sec.
- The value of maximum phase lead $\phi_{max} = \text{Sin}^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$



Effects of lead compensators:

Advantages:

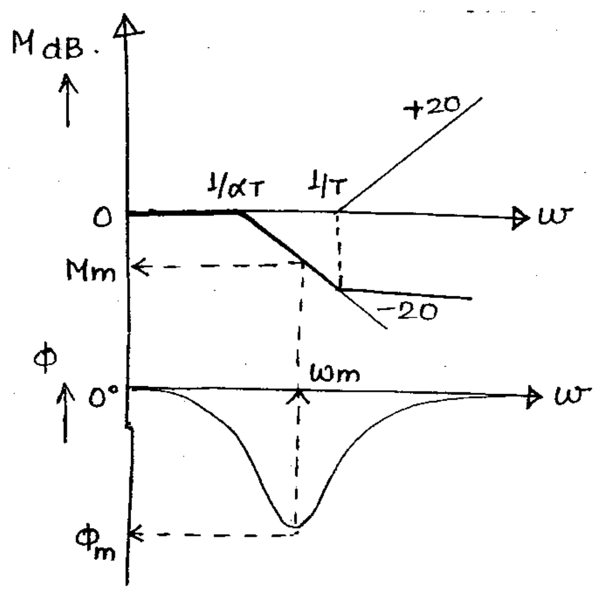
- The lead compensator is a high pass filter hence the bandwidth of the system increases.
- As bandwidth increases the rise time decreases. Hence the transient performance improved.
- The lead compensator improves the damping of the system hence settling time decreases.
- The lead compensator improves the phase margin and gain margin hence relative stability is improved.
- The lead compensator is similar to PD-controller.

Disadvantages:

- The lead compensator is a high pass filter hence the noise is enter into the system. So the signal to noise ratio at the output is poor.
- In lead compensator we required to add a amplifier which adds cost and space.
- The maximum phase lead given by the lead compensator is 60^0 if required more than 60^0 then we required to use multi section compensators.

Lag Compensators:

- When a sinusoidal input is applied to a network it produce the sinusoidal steady state output having phase lag with respect to input, then the network is called lag compensator.
- The lag compensator improves the steady state behaviour (Steady state error decreases).
- The frequency at which minimum phase lag occurs $\omega_m = \dfrac{1}{T\sqrt{\beta}}$ rad/sec.
- The value of minimum phase lag $\phi_{min} = \text{Sin}^{-1}\left(\dfrac{\alpha - 1}{\alpha + 1}\right)$



Effects of Lag Compensators:

Advantages:

- The lag compensator is a low pass filter hence the noise is filtered out. So the signal to noise ratio at the output is improved.
- The lag compensator improves the steady state behaviour.
- The lag compensator similar to PI-controller.

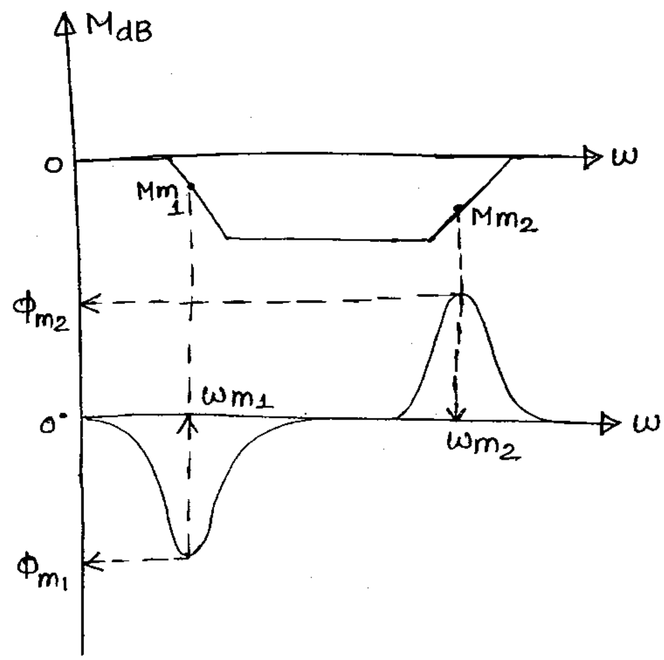
Disadvantages:

- The lag compensator is a low pass filter hence the bandwidth of the system is decreased.
- As bandwidth decreases the rise time increases.

- In lag compensator the attenuation characteristics are used for the compensation, where as there is no used phase lag characteristics for compensation.
- With lag compensator the system becomes very sensitive with parameter variation.

Lag Lead Compensator:

- The lag-lead compensator is used to get the very quick response and good static accuracy.



CONTROLLERS:

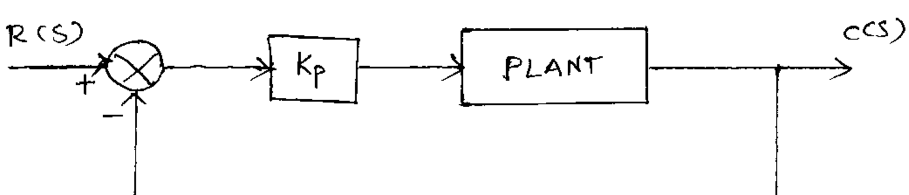
- A controller is a device which is used to change the transient and steady state performance as per the requirement.
- The best system is the one it should have lowest rise time, smallest settling time, smallest steady state error, smallest peak over shoot to get above requirement to use the controller.
- The block diagram with the controller is shown in figure.

Types of Controllers:

1. Proportional Controller:
2. Integral Controller
3. Derivative Controller
4. Proportional plus Integral Controller
5. Proportional and Derivative Controller
6. PID Controller

PROPORTIONAL CONTROLLER

Purpose : To change the transient response as per the requirement

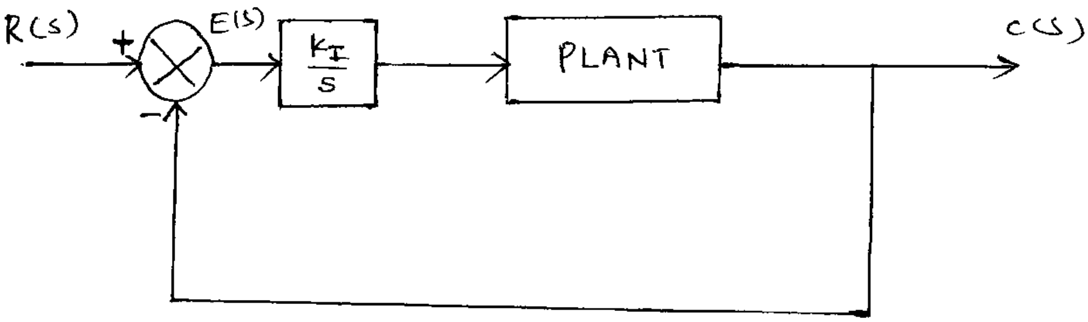


Disadvantage:

- The p-controller can’t eliminate the complete error in the system.
- As the K_p value increases the damping ratio ‘ξ’ decreases, hence % peak over shoot increases so the system becomes more oscillatory and less relatively stable.

INTEGRAL CONTROLLER

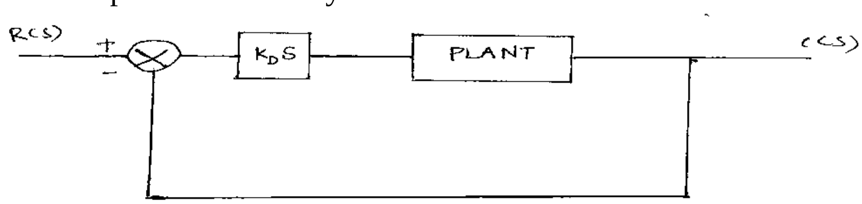
Purpose: To decrease the steady state error



- The Transfer Function of I-controller is $\frac{K_I}{S}$
- The I-controller added one pole at origin which increases the type of the system. As type increases steady state error decreases but the system stability is effected.
- Before using the integral controller we require to check the system stability. If the stability is effected then the I-controllers are not used.

DERIVATIVE - CONTROLLER

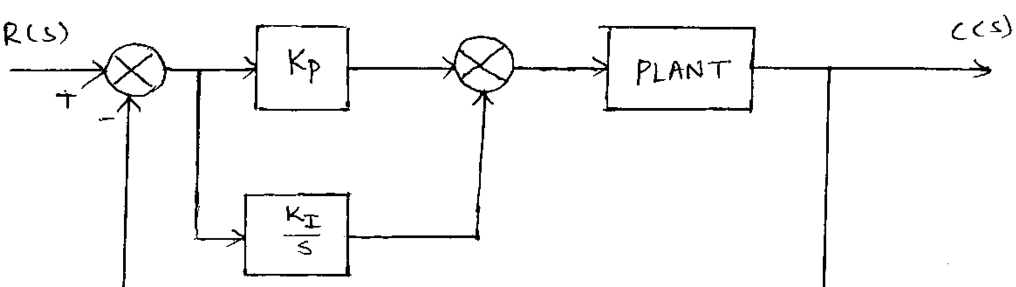
Purpose: To improve the stability



- The Transfer Function of derivative controller is $K_D.S$
- The best example for D-controller is tachometer.
- The D-controller added one zero at origin hence the type of the system decreases.
- As type decreases the stability improved but the steady state error increases so the system becomes less accurate.

PROPORTIONAL PLUS INTEGRAL CONTROLLER

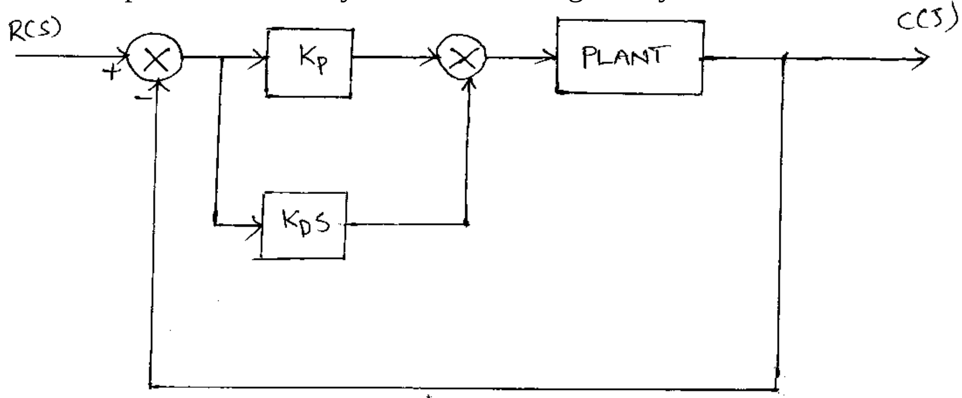
Purpose: To decrease the steady state error without effecting stability



- The Transfer Function of PI-Controller is $\left(K_p + \frac{K_I}{S}\right)$
- The PI-controller added one pole at origin hence the type of the system increases. As type increases steady state error decreases.
- The PI-controller added one finite zero in the left hand side, which avoids the effect on system stability due to the addition of pole at origin.

PROPORTIONAL AND DERIVATIVE CONTROLLER

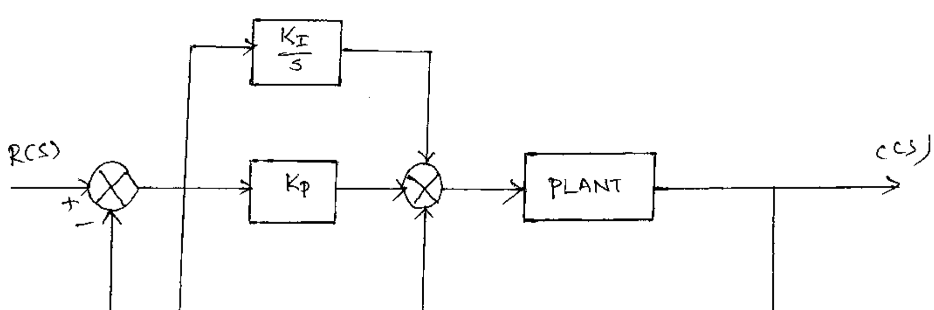
Purpose: To improve the stability without effecting steady state errors.



- The Transfer Function of PD-controller is $(K_p + K_D S)$
- The PD-controller added one finite zero in the left hand side which improves the system stability.
- The PD-controller not changes the type hence no effect on steady state error.

PID CONTROLLER

Purpose: To improve the stability and decreases the steady state error.



- The PID – controller added one pole at origin, hence the type of the system increases. As type increases steady state error decreases.
- The PID controller added two finite zeros, in the left hand side, one finite zero, avoid the effect on stability. And another zero improves the stability.

IMPORTANT POINTS:

- The transfer function is defined as the ratio of Laplace transform of output to Laplace transform of input under assumption that all initial conditions are zero.
- The stability of a time-invariant system can be determined from the characteristic equation. Consequently, for continuous systems, if all the roots of the denominator have negative real parts, the system is stable.
- The system differential equation can be obtained from the transfer function by replacing the **s** variable with **d/dt**.
- Transfer function is valid only for linear time invariant system.
- The value of **s** for which the system magnitude [G(s)] becomes infinity are called poles of G(s). When pole values are not repeated, such poles are called as simple poles. If repeated such poles are called multiple poles of order equal to the number of times they are repeated.
- The value of s for which the system magnitude [G(s)] becomes zero are called zeros of transfer function G(s). When they are non repeated, they are called simple zero, otherwise they are called multiple zeros.
- Type : The number of poles at the origin in an Open Loop transfer function is called as Type of the system.
- Order : The highest power in the denominator polynomial of Closed Loop control system is defined as the Order.
- In Block diagram reduction, gain of the blocks in series gets multiplied whereas that of in parallel gets added or subtracted depending upon the sign of the summer.
- Signal flow graph specifications :
  i) The node having only outgoing branches is known as source or input node.
  ii) The node having only incoming branches is known as sink or output node.
  iii) A node having incoming and outgoing branches is known as chain node.
  iv) A path from an input to an output node is defined as forward path.
  v) A loop which originates and terminates on the same node is known as feedback path.
  vi) A feedback loop consisting of only one node is called self loop.
  vii) A self loop cannot appear while defining a forward path or feedback path as node containing it gents traced twice which is not allowed.
  viii) The product of branch gains while going through a forward path is known as path gain. This can also be called forward path gain.

ix) If there exists incoming and outgoing branches both at 1st and last node,l representing input and output variables, then as per definition these cannot be called as source or sink nodes. In such a case separate input and output nodes can be created by adding branches with gain 1. Such nodes are called as Dummy nodes.

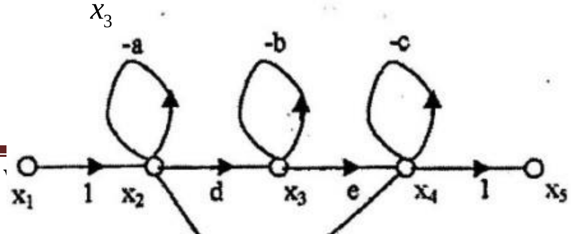
- Mason's formula :

$$TF = \sum_{K=1}^{n} \frac{P_K \cdot \Delta_K}{\Delta}$$

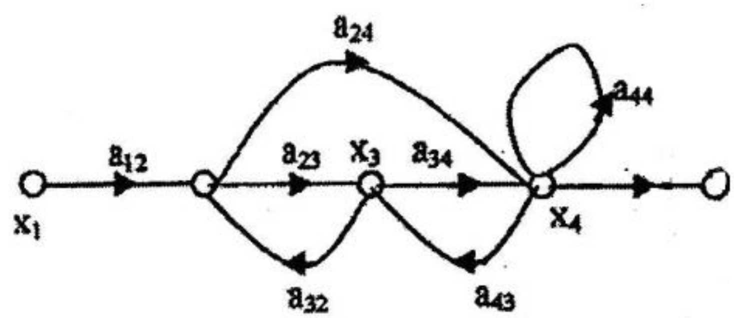
- A device inserted into the system for the purpose of satisfying the specifications is called a compensator.
- If a sinusoidal input l_i is applied to the input of a network and the steady state output l_0 has a phase lead, then the network is called lead network.
- If the steady state output e_0 has a phase lag, then the network is called a lag network.
- The medium frequency region of the locus indicates relative stability. The high frequency region indicates the complexity of the system.
- Lead network has increased bandwidth, increased damping ratio and improved phase Margin.
- The minimum value of α is limited by the physical construction of the lead compensator. The minimum value of α is usually taken to be about 1/12.
- The phase lag angle does not play a role in lag compensation. Attenuation at high frequency is used for compensation.
- The lag compensation decreases the bandwidth of the system, but increases stability.
- The lag network has a dc gain of unity while it offers a high frequency gain of $1/\beta$.
- Lead compensator raises the order of the system by one.
- Zero is located to the right of pole and nearer to the origin.
- Lag compensator is basically a low pass filter. Thus it allows high gain at low frequencies and improves the steady state response.
- The use of lead or lag compensators raises the order of the system by one. The use of lag-lead compensator raises the order of the system by two.
- The P controller changes the transient response as per requirement but it does not eliminate the error in the system.
- The I controller reduces the steady state error but the
- Proportional plus derivative controller reduces the peak overshoot and settling time.
- As PD controller improves transient part and PI controller improves steady state part; thus, overall time response of the system improves drastically in case of PID Type Controller.

PRACTICE QUESTIONS:

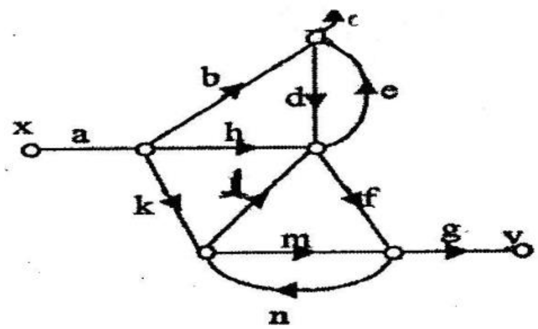
1. Find 1) $\frac{x_5}{x_1}$ 2) $\frac{x_5}{x_3}$



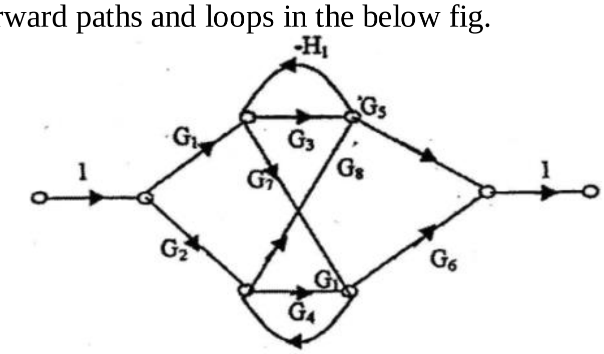
2. Find $\frac{x_4}{x_1}$ *and* $\frac{x_3}{x_1}$



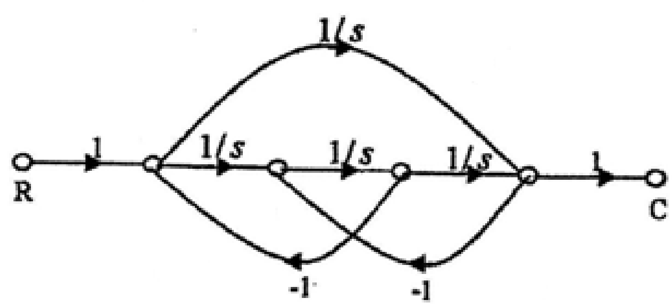
3. In the figure below find the no.of forward paths and loops



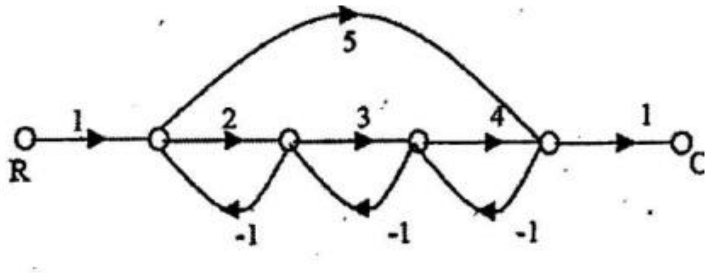
4. Find the no.of forward paths and loops in the below fig.



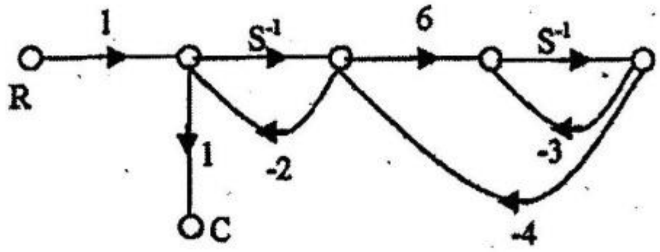
5. Find C/R



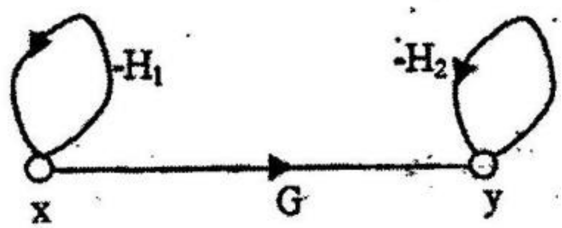
6. C/R of the fig is



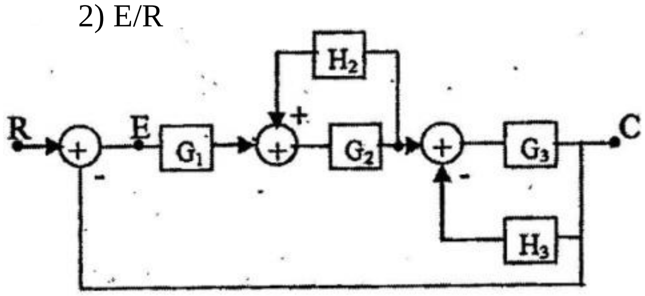
7. Find C/R



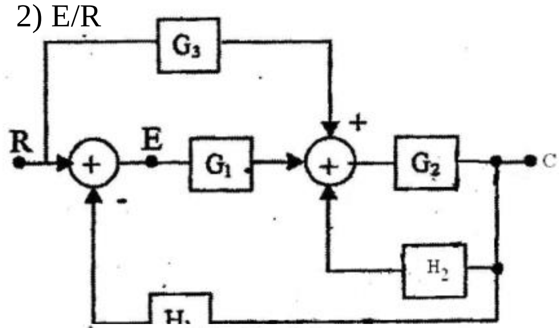
8. Find y/x



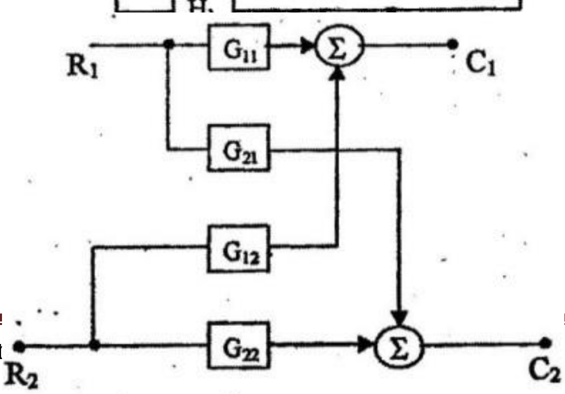
9. Find 1) C/R 2) E/R



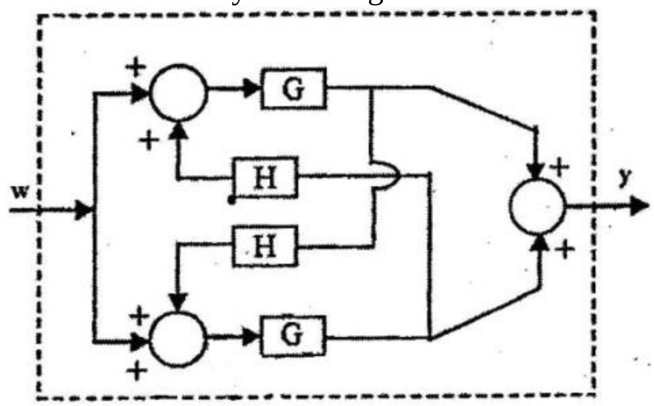
10. Find 1) C/R 2) E/R



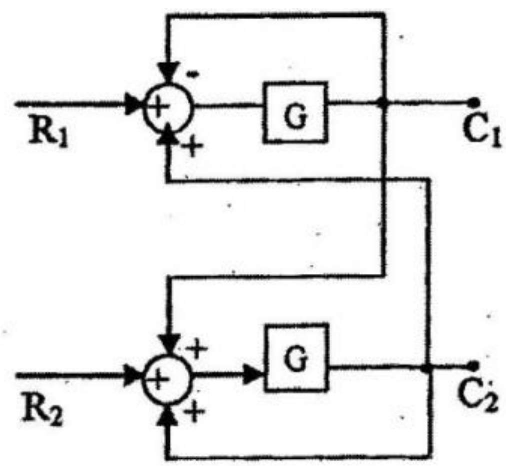
11. Find C_1&C_2



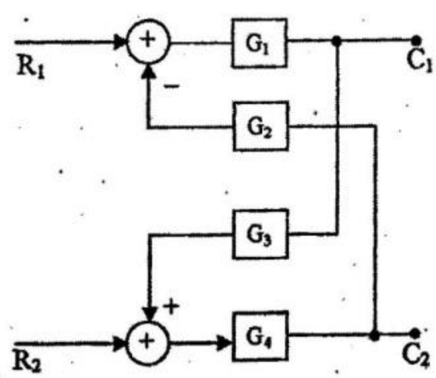
12. The overall transfer function of the system in figure is



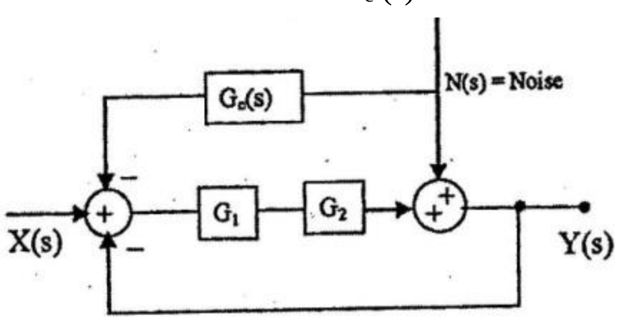
13. Find C_1 and C_2



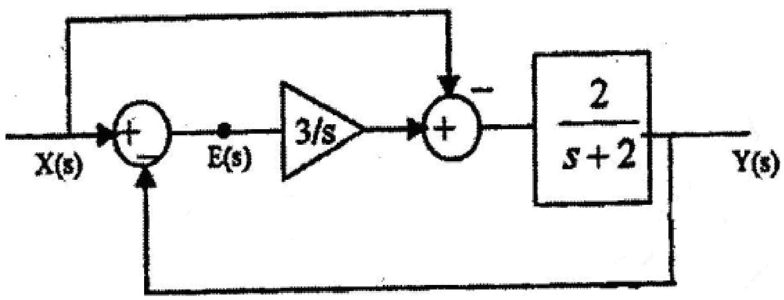
14. Find C_1&C_2



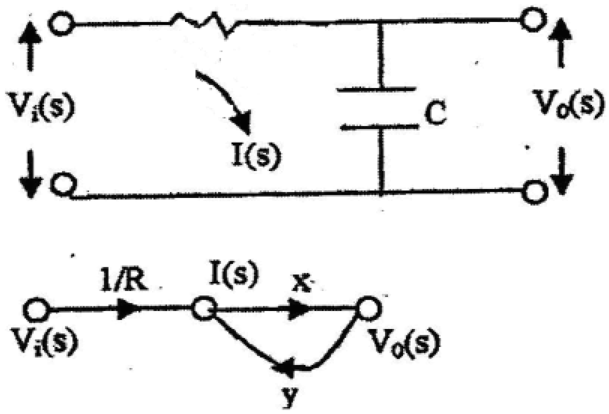
15. For the system shown find the condition on G_c (s) in order to nullify the noise at the output.



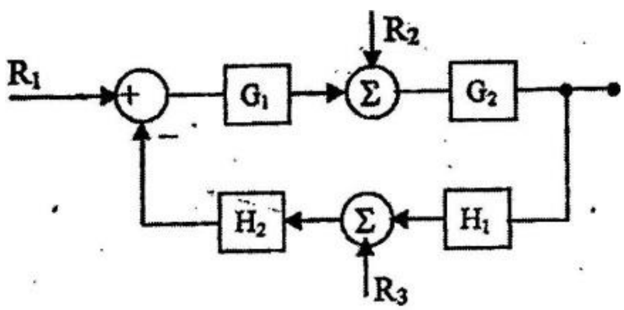
16. Find i) $\frac{Y(S)}{X(S)}$ ii) $\frac{E(S)}{X(S)}$



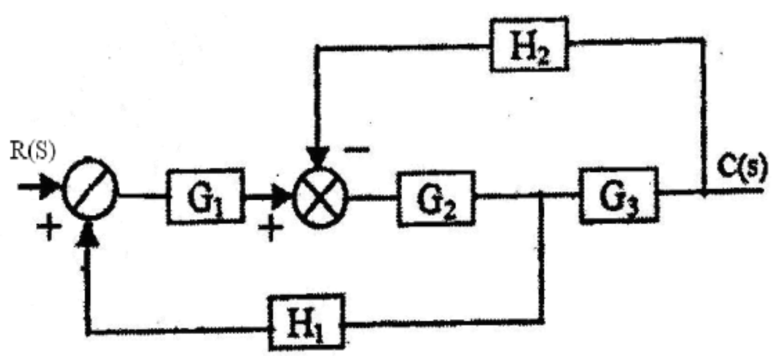
17. The electrical n/w and its equivalent SFG is given below Find the values of x and y



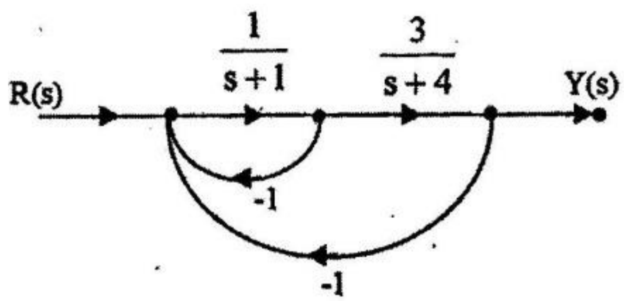
18. Find the value of 'C' and C/R_3 in the following figure.



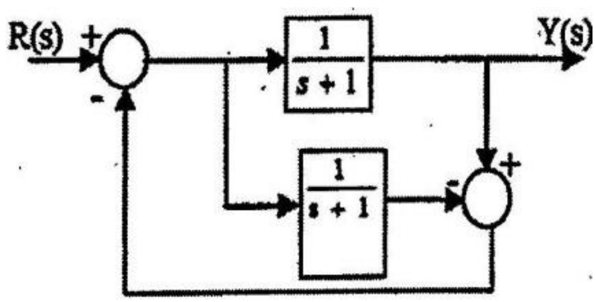
19. For block diagram shown in figure, find C(s)/R(s)



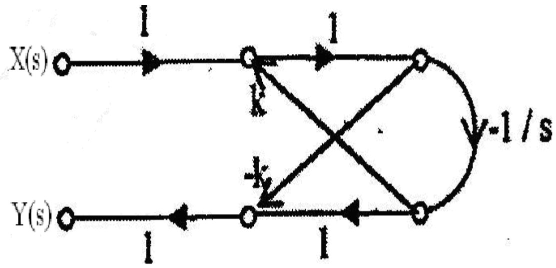
21. For the flow diagram shown in Fig. below, find the transfer function $\frac{Y(s)}{R(s)}$



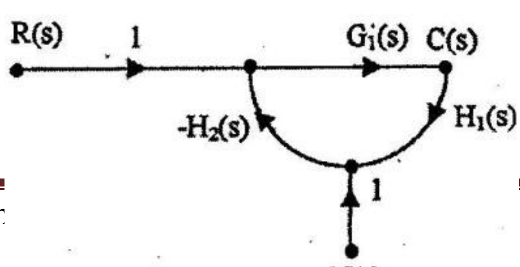
22. Find the transfer function Y(s)/R(s) of the system shown



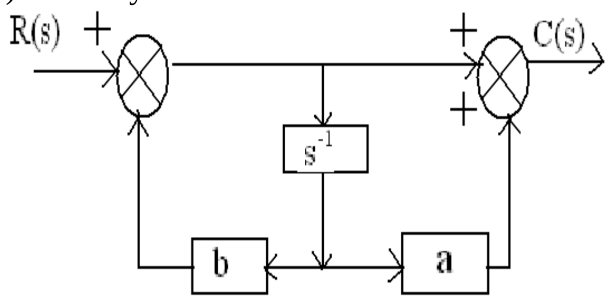
23. A filter is represented by the signal flow graph shown in the figure. Its input is x (t) and output is y (t). Find the transfer function of the filter



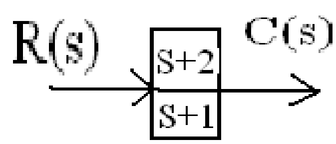
24. A closed-loop system is shown in the given figure. Find the noise transfer function C (s)/ N(s) [C_n (s) = output corresponding to noise input N (s) is approximately.

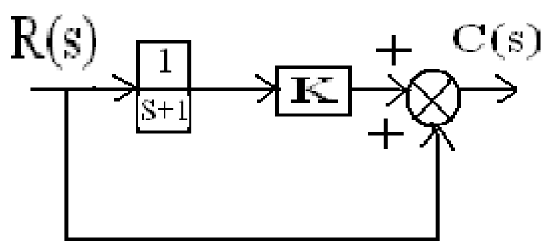


25. The block diagram for a particular control system is shown in the figure. What is the transfer function C (s) / R(s) for the system?

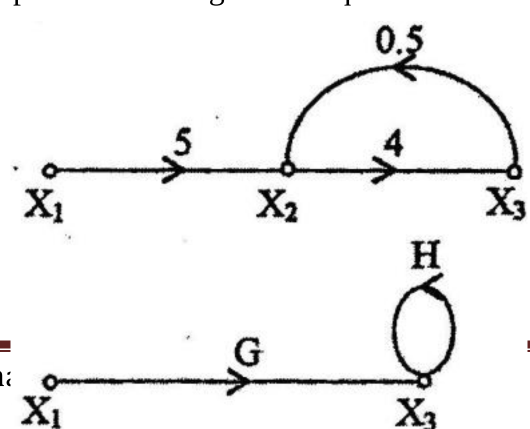


26. For what value of K are the two block diagrams as shown below are equivalent

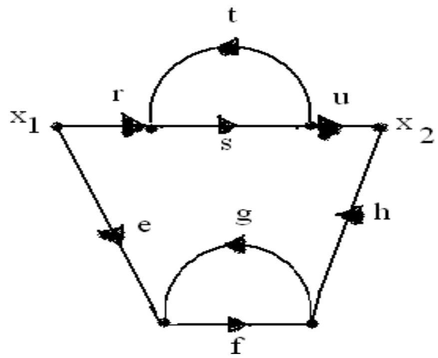




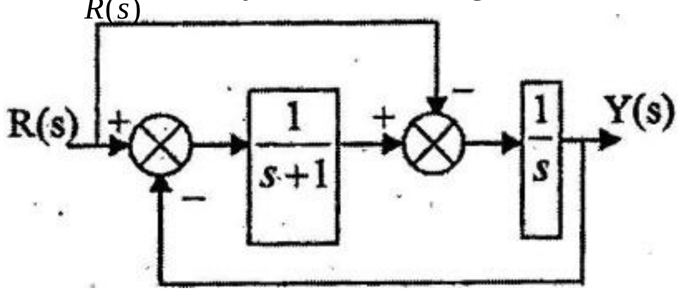
27. The two signal flow graphs shown in figure are equivalent. Find the value of G and H ?



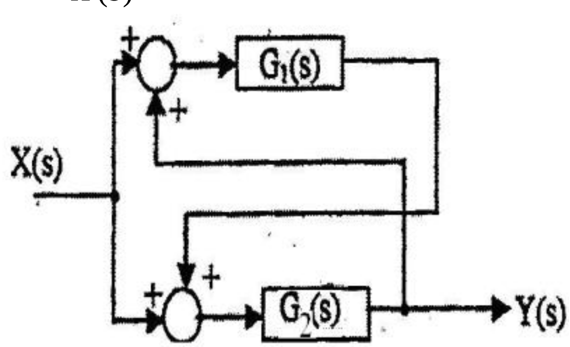
28. For the signal flow graph shown in the given figure, find the transmittance between X_2 and X_1 is



29. Find the transfer function $\frac{Y(s)}{R(s)}$ of the system shown in fig. is

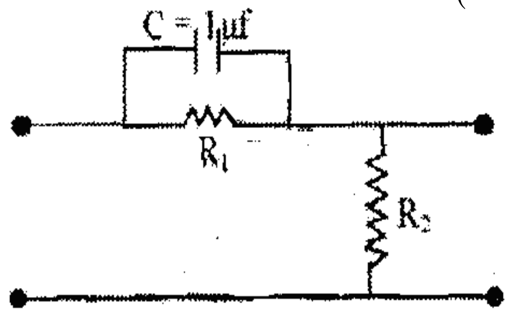


30. Find the transfer functions $\frac{Y(s)}{X(s)}$ of the linear time invariant system shown in Fig.



31. Find the Maximum phase lead of the compensator $G_C(s) = \frac{1+0.5S}{1+0.25S}$

32. The TF of a lead network as shown in figure below is $\frac{K(1+0.3S)}{(1+0.17S)}$. Find the values of R_1 & R_2 are



UNIT – III

TIME RESPONSE ANALYSIS

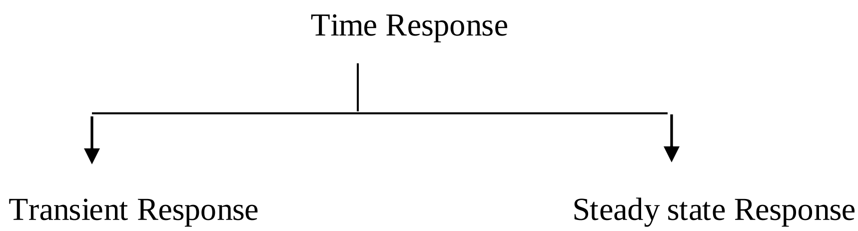
- Before proceedings with the time response analysis of a control system; it is necessary to test the stability of the system.
- Time response is the output variation of the system with respect to time by applying the standard test signals.
- The response of the system as a function of time, given in response to the applied excitation is called Time Response. The time response of a system can be fully studied by studying the following two responses.

TIME RESPONSE

The time response of the system is the output of the closed loop system as a function of time.

The response of the control system is divided into two parts :

1. Transient Analysis
2. Steady State Analysis



Mathematically,

The total time response C(t) is given by $C(t) = C_{tr}(t) + C_{ss}(t)$

Where $C_{tr}(t)$ = Transient response

$C_{ss}(t)$ = steady state response

TRANSIENT RESPONSE (NATURAL RESPONSE) :

It is defined as that part of the time response that goes to zero as time becomes very large i.e. input and output responses as a function of time

$$\lim_{t \to \infty} C(t) = 0$$

- The transient response also called "Dynamic Response" of the system
- The transient response depends on the system poles only not on the type of input (step, ramp, parabolic, impulse)
- The transient part of time response gives the nature of response (i.e. oscillatory (or) over damped) and also gives an indication about its speed.
- The transient response depends on the order of the system.

STEADY STATE (OR) ZERO STATE (OR) FORCED (OR) FIXED (OR) FINAL (OR) PARTICULAR RESPONSE:

- It is that part of the response which is fixed when time 't' tends to ∞ .

- Here the steady state accuracy of a system is analyzed.
- It is a part of the time response which deals with the calculation of error.

$$\frac{C(s)}{R(s)} = TF$$

C(s) = [(TF) R(s)]

Time response c(t) = L^{-1}[C(S)]

Initial value of the response [C(0)] :

$$C(0) = \lim_{\tau \to 0} C(t) = \lim_{S \to \infty} SC(s)$$

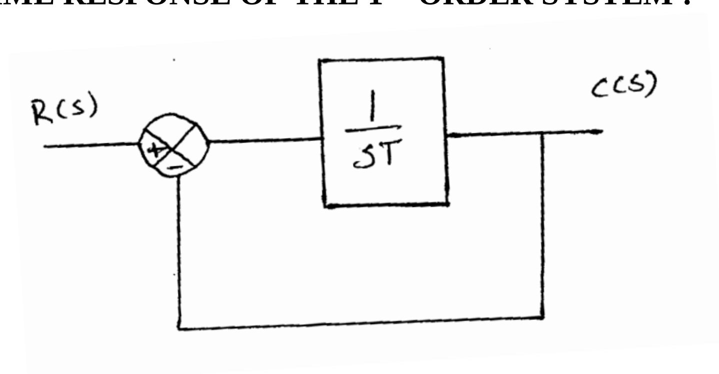
Initial value theorem is applied only when the number of poles of C(S) is more than the number of Zero's i.e. the function C(S) must be strictly proper.

Final value of the response $[c(\infty)]$**:**

$$C(\infty) = \lim_{t \to \infty} C(t) = \lim_{S \to 0} SC(s)$$

Final value theorem is applied when all the poles of C(s) lie in the left half of the S-plane.

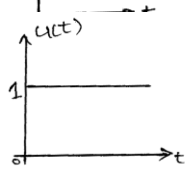
TIME RESPONSE OF THE 1ST ORDER SYSTEM :



$$\frac{C(s)}{R(s)} = \frac{1}{1+TS}$$

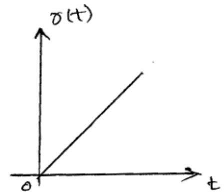
Where T = time constant

SIGNAL NAME	SIGNAL	LAPLACE TRANSFORM
Unit Impulse		1

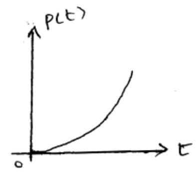


Unit step: $\frac{1}{S}$

Unit ramp: $\frac{1}{S^2}$



Unit Parabolic: $\frac{1}{S^3}$



UNIT IMPULSE RESPONSE (IR) :

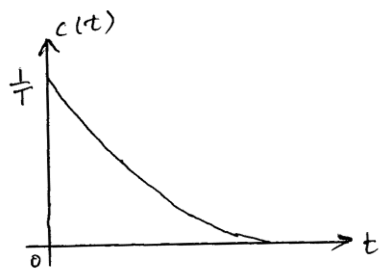
r(t) = $\delta(t)$

R(S) = 1

C(S) = $\frac{1}{1+ST}$

Apply Inverse laplace transform

c(t) = $\frac{1}{T}e^{-t/T}$



Note :

- The impulse response consists only transient terms. Hence it is called system response (or) zero input response (or) natural response (or) free forced response.
- Steady state error cannot be defined for impulse input because it has only transient term.

UNIT STEP RESPONSE :

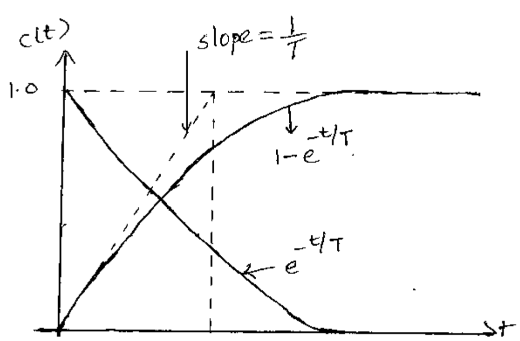
R(S) = $\frac{1}{S}$

$C(S) = \frac{1}{1+TS}$;

C(s) = $\frac{1}{S} - \frac{T}{1+TS}$

Apply inverse Laplace transform c(t) =L^{-1}[C(s)]

c(t) = 1- $e^{-t/T}$



The output raises exponentially from 0 to the final value of unity.
The initial of curve at t = 0 is given by

$$\left.\frac{dc}{dt}\right|_{t=0} = \left.\frac{1}{T}e^{-t/T}\right|_{t=0} = 1/T$$

Where T is known as time constant of the system.

TIME CONSTANT :
The time required for response reaches to 63.2% of its final value.
The time constant is indicative of how fast the system tends to reach the final value.

STEADY STATE ERROR :

$$e(t) = r(t) - c(t) = e^{-t/T}$$

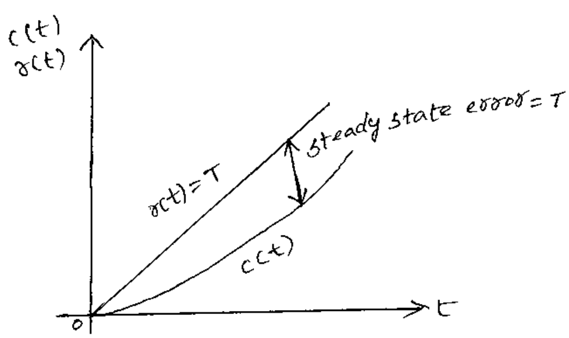
$$e_{ss} = \lim_{t\to\infty} e(t) = 0$$

where e_{ss} = steady state error
Thus the system tracks the unit step input with zero steady state error

UNIT RAMP RESPONSE:
The output response for the unit ramp input R(S) = $\frac{1}{S^2}$ is given,

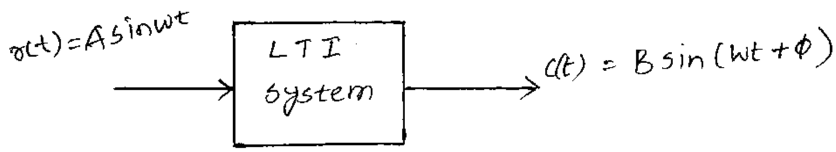
$$C(S) = \frac{1}{S^2(1+ST)} = \frac{1}{S^2} - \frac{T}{S} + \frac{T^2}{1+ST}$$

Taking inverse laplace transform
c(t) = t - T$(1-e^{-t/T})$
The error signal is, e(t) = r(t) – c(t) = T$(1-e^{-t/T})$ and
the steady state error is, $e_{ss} = \lim_{t\to\infty} e(t) = T$
Thus the first order system under consideration will track the ramp input with steady state error T, which is equal to the time constant of the system.
NOTE :
Reducing the system time constant therefore not only improves its speed of response but also reduces its steady state error to a ramp input.



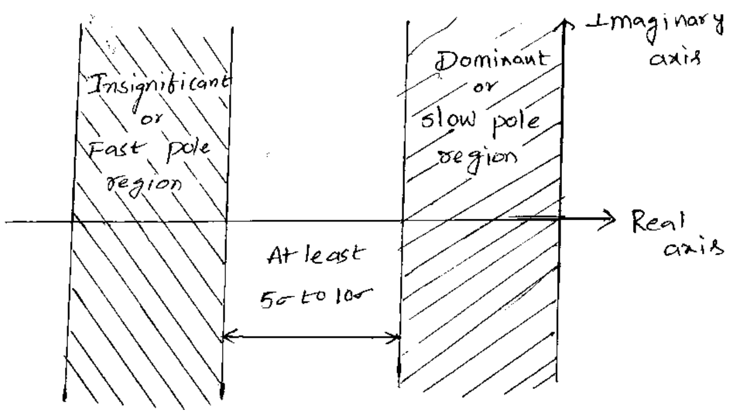
SINUSOIDAL RESPONSE:

Sinusoidal input is given to any linear time invariant system the output also a sinusoidal but the difference in magnitude and phase.



DOMINANT (OR) SLOW POLES :

The time constant of the poles nearer to the imaginary axis is greater than the poles that are farther from the imaginary axis. Since the time constant is larger these poles are called as slow poles . The slow poles dominate the transient response, hence these poles are called as dominant poles .As far as the speed of the response is considered fast poles are ignored hence called as insignificant poles.



STANDARD (OR) PROTOTYPE SECOND ORDER SYSTEM

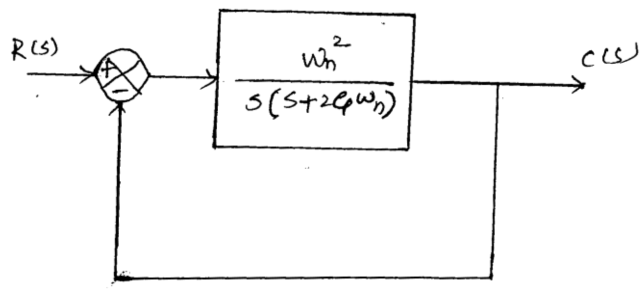


Fig. : Second Order System

$$TF = \frac{C(S)}{R(S)} = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

Where

ξ = Damping ratio

ω_n = Un damped natural frequency

The time response of any system is characterized by the roots of the denominator polynomial which infact are the poles of the transfer function.

The denominator polynomial is therefore called the characteristic polynomial and is called the characteristic equation

$S^2 + 2\xi w_n s + w_n^2 = 0$ is called as the characteristic Equation.

Roots are $S_{1,2} = -\xi w_n \pm j\,\omega_n \frac{2s+1}{s^2}$

$= -\alpha \pm j\omega_d$

Where $\alpha = \xi w_n$ is called as Damping factor /coefficient /constant

ω_d = damped natural frequency

- α controls the rate of rise or decay of the unit step response
- α controls the damping of the system and is called damping factor or damping constant.
- The inverse of α is called the time constant of the system.

STEP RESPONSE OF THE STANDARD SECOND ORDER SYSTEM

R(S) = $\frac{1}{S}$

Case (i) : Under Damped System $(0 < \xi < 1)$ **:**

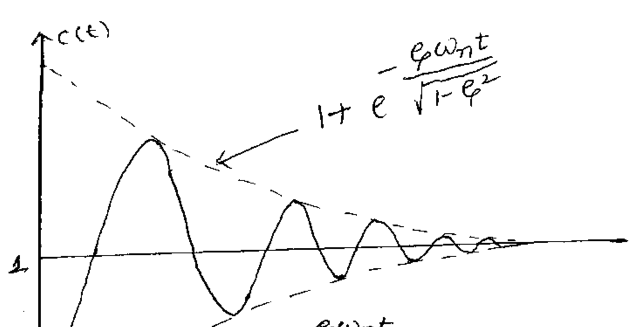
We know that TF = $\frac{C(S)}{R(S)} = \frac{w_n^2}{S^2 + 2\xi w_n S + w_n^2}$

$$\frac{C(S)}{R(S)} = \frac{w_n^2}{S^2 + 2\xi w_n S + w_n^2} R(S)$$

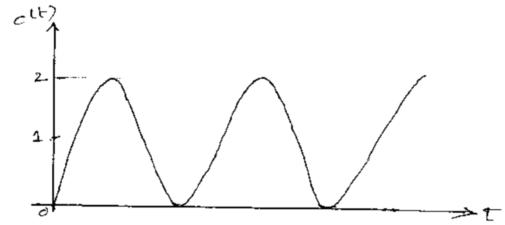
$$C(S) = \frac{w_n^2}{S^2 + 2\xi w_n S + w_n^2}\frac{1}{S}$$

Taking Inverse Laplace transform c(t) = L^{-1}[C(S)]

$$C(t) = 1 - \frac{e^{-\xi w_n t}}{\sqrt{1-\xi^2}} \text{ Sin } (\omega_d t + \phi)$$



Case (ii) : Un Damped System ($\xi = 0$) :

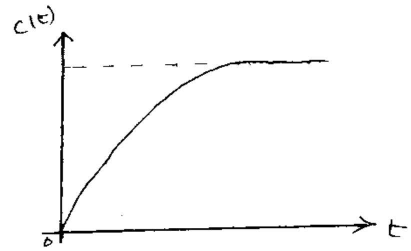


Substitute $\xi = 0$ in above equation,

c(t) = $1 - \sin(\omega_n t + \pi/2)$

c(t) = $1 - \cos\omega_n t$

Case (iii) : Critically Damped System ($\xi = 1$) :



$$C(S) = \frac{\omega_n^2}{S(S^2 + 2\omega_n S + \omega_n^2)} = \frac{\omega_n^2}{S(S+\omega_n)^2}$$

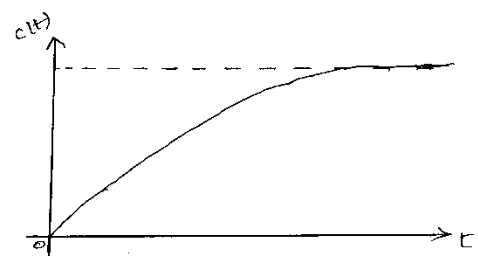
c(t) = $1 - e^{-\omega_n t} - \omega_n t.e^{-\omega_n t}$

Case (iv) : Over Damped System ($\xi > 1$)

$$C(S) = \frac{\omega_n^2}{S(S^2 + 2\omega_n S + \omega_n^2)}$$

$$= \frac{\omega_n^2}{S(S + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1})(S + \xi\omega_n - \omega_n\sqrt{\xi^2 - 1})}$$

$$= \frac{1}{S} - \left[\frac{A}{(S + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1})} + \frac{B}{(S + \xi\omega_n - \omega_n\sqrt{\xi^2 - 1})}\right]$$



c(t) = $1 - \left[A.e^{(\xi\omega_n + \omega_n\sqrt{\xi^2 - 1})t} - B.e^{(\xi\omega_n - \omega_n\sqrt{\xi^2 - 1})t}\right]$

POLE LOCATIONS FOR SECOND ORDER SYSTEM :

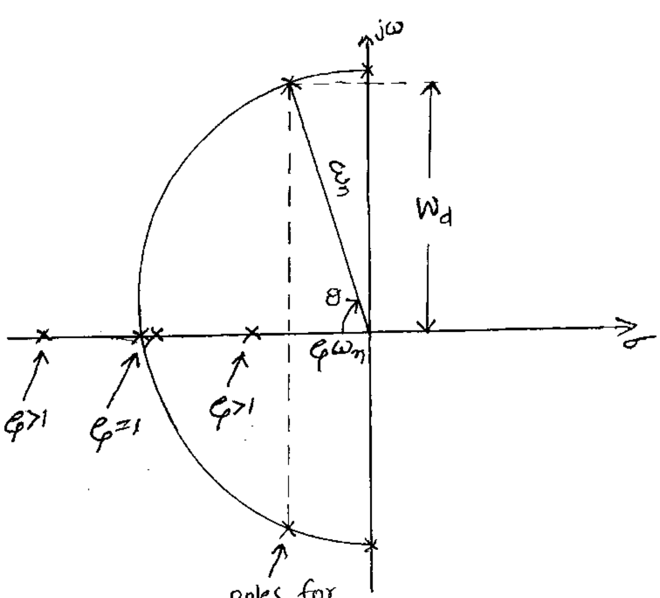
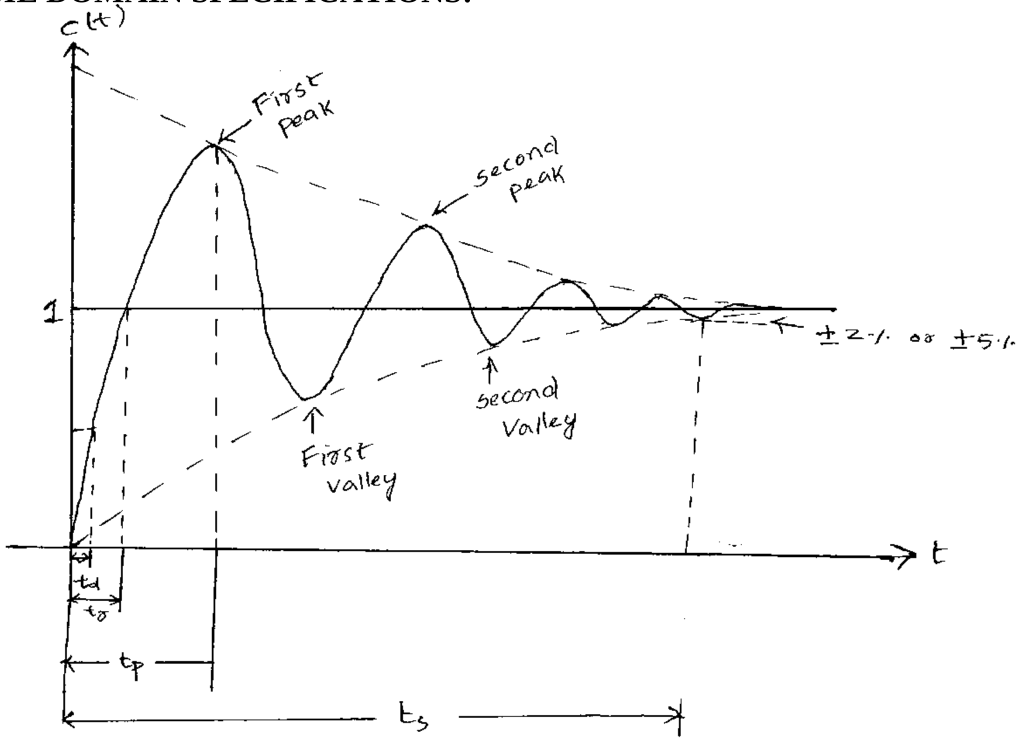


Fig. : Locus of the poles of the second order system.

- As 'ξ' increases from 0 to 1, the poles moves left and nearer to real axis hence the system time constant decreases and the response progressively less oscillatory till it becomes critically damped. (just non oscillatory) for ξ.
- ω_n is the radial distance from the roots to the origin.
- α is the real part of the roots. ω_d is the imaginary part of the roots.
- $\xi = \cos\theta$, ξ is the cosine of the angle between the radial line to the roots and the negative axis when the roots are in the left-half of s-plane
- When $\xi > 1$ and increases, the system time constant increases because one pole move towards the origin on the real axis.

TIME DOMAIN SPECIFICATIONS:



Delay time (t_d) :

It is the time required for the response to reach 50% of the final value in first attempt

$$t_d = \frac{1 + 0.7\xi}{w_n}$$

Rise time (t_r) :

It is the time required for the response to rise from 10% to 90% of the final value for over damped systems and 0 to 100% of the final value for the under damped systems.

$$t_r = \frac{\pi - \phi}{w_d}$$

Peak time (t_p) :

It is the time required for the response to reach the peak of time response or the peak overshoot.

$$t_p = \frac{n\pi}{\omega_d}$$ n = 1 for first peak

n = 2 for first valley

- Time period of damped oscillations is equal to the twice of the peak time = $2t_p$

Maximum (or) Peak overshoot (M_p) :

It indicates the normalized difference between the time response peak and steady output and is defined as

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} * 100\%$$

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

In most control systems (except type-0), the steady output for step input is the same as the input.

Settling time (t_s) :

It is the time required for the response to which and stay within a specified tolerance band (usually 2% or 5%) of its final value

$$t_s \approx \frac{3}{\xi\omega_n}$$ for 2% tolerance band

$$= t_s \approx \frac{4}{\xi\omega_n}$$ for 5% tolerance band

- If ξ increases , rise time, peak time increases and peak overshoot decreases .
- Number of oscillations before reaching steady state,

$$N = \frac{Total\ time\ required}{one\ oscillation\ time}$$

$$= \frac{t_s(\pm 2\% or \pm 5\%)}{T_{osc}} = \frac{t_s}{2t_p}$$

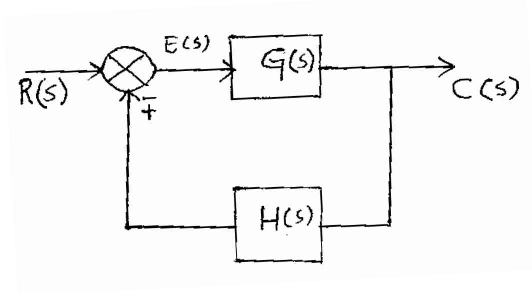
- Time period of oscillations : It is the time required to complete one cycle.

$$T_{osc} = \frac{2\pi}{\omega_d} \sec$$

✓ Why under damped system is preferable ?

For quick response i.e. small rise time and smallest settling time, the control system should be under damped. So control systems are normally designed with $\xi < 1$

STEADY STATE ERROR AND STATIC ERROR CONSTANTS :



E(s) = Error signal

R(s) = Reference input (or) Designed output

C(s) = actual output

$$\frac{E(S)}{R(S)} = \frac{1}{1+G(S)} \qquad \therefore E(S) = \frac{1}{1+G(S)} R(S)$$

STEADY STATE ERROR (E_{SS}) : It is the deviation from the desired output.

e(t) = r(t) – c(t)

$e_{ss} = \lim_{t\to\infty} e(t)$

$e_{ss} = \lim_{S\to 0} SE(S)$

$e_{ss} = \lim_{S\to 0} S(R(S) - C(S))$

$e_{ss} = \lim_{S\to 0} \frac{SR(S)}{1+G(S)}$

Steady state error to a step input :

R(S) = $\frac{A}{S}$

A = Amplitude of the step input

$e_{ss} = \lim_{S\to 0} SE(S) = \lim_{S\to 0} S\left[\frac{1}{1+G(s)} . \frac{A}{S}\right]$

$e_{ss} = \frac{1}{1+\lim_{S\to 0} G(s)} = \frac{A}{1+K_p}$

Where $K_p = \lim_{s\to 0} G(S)$ is known as static position error coefficient (or) constant.

Steady state error to a Ramp input :

R(S) = $\left(\frac{A}{S^2}\right)$

Similarly $e_{ss} = \frac{A}{K_v}$

$K_v = \lim_{S\to 0} S(G(s))$ is known as static velocity error coefficient.

Steady state error to a parabolic input :

R(S) = $\frac{A}{S^3}$

Similarly $e_{ss} = \dfrac{A}{K_a}$

Where $K_a = \lim_{S \to 0} S^2 G(s)$ is known as static acceleration error coefficient .

TYPE	Steady state error		
	Step $= \dfrac{A}{S}$	**Ramp** $= \dfrac{A}{S^2}$	**Parabolic** $= \dfrac{A}{S^3}$
0	$\dfrac{A}{1+K_P}$	∞	∞
1	0	$\dfrac{A}{K_V}$	∞
2	0	0	$\dfrac{A}{Ka}$
3	0	0	0

Note :

i) The error constants for non unity feed back systems may be obtained by replacing G(S) by G(S) . H(S).

ii) If the type of a system increases the error decreases , accuracy increases but the stability decreases.

Disadvantages of static error constants :

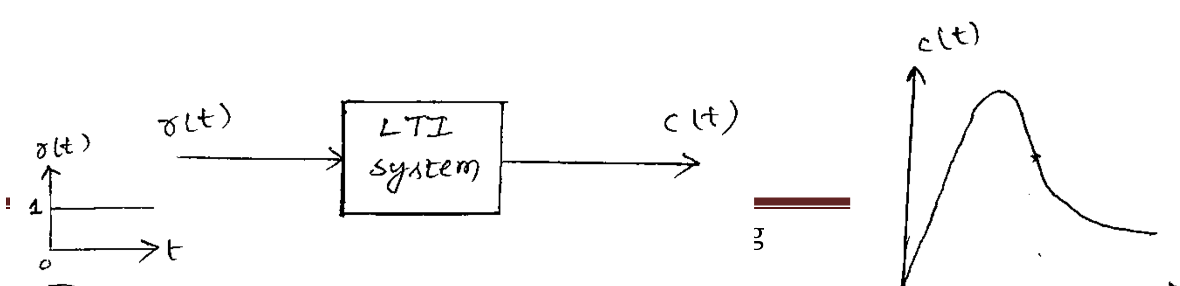
1. It cannot give error if inputs are other than the standard test inputs
2. It cannot give precise value of error
3. It does not provide variation of error with respect to time.

STABILITY ANALYSIS

STABEL SYSTEM :

A linear time invariant system is said to be stable if following conditions are satisfied.

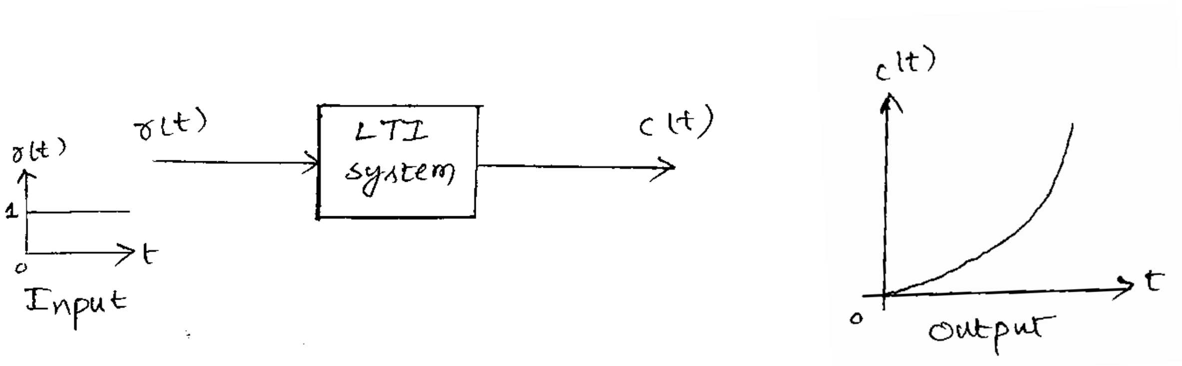
1. When the system is excited by a bounded input, output is also bounded and controllable
2. In the absence of the input, output must tend to zero irrespective of the initial conditions



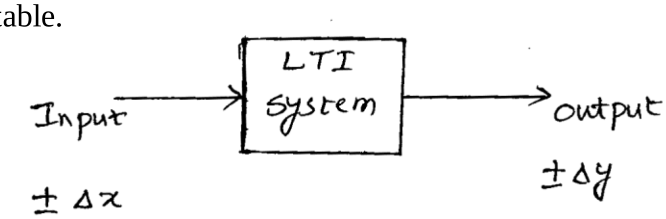
UNSTABLE SYSTEM :
A linear time invariant system is said to be stable if,

1. For a bounded input it produces unbounded output
2. In the absence of input, output may not return to zero. It shows certain output without input.

Physically an unstable system whose natural response grows without bound can cause damage to the system, to adjascent property or to human life.



STABILITY : To an LTI system if input is bounded and output is bounded then the system is said to be stable.

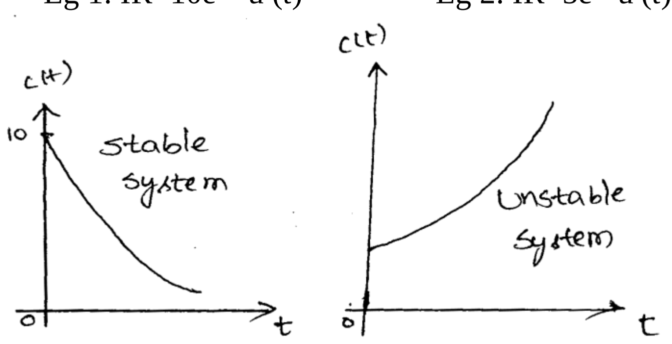


If $0 < |\Delta x| < \infty$ then $0 < |\Delta y| < \infty$
BI BO

CONDITION FOR STABILITY:
The IR of a stable system must approach to zero, when time $t \rightarrow \infty$
(or) The IR of a stable system must be absolutely integrable between the limits 0 and ∞.

i.e., $\int_0^\infty |IR| dt < \infty$

Eg 1: IR=$10e^{-3t}$ u (t) Eg 2: IR=$5e^{2t}$ u (t)



Eg 1: IR= $10e^{-3t}$ u (t)

$\int_0^\infty 10e^{-3t} = \frac{10}{3}$ Stable, Eg2: IR=$5e^{2t}$ u (t)

$\int_0^\infty 5e^{2t} = \infty$ Not integrable, Hence unstable

The above two conditions are time domain conditions, the S-domain condition is all the poles of a system should lie in the left half of‘s’ plane i.e., poles should have negative real parts.

Eg1: TF=$\frac{10}{s+3}$; Stable, Eg2: TF=$\frac{5}{s-2}$; Unstable

- The nature of impulse response g(t) is depend on the poles of transfer function G(S) which are the rules of the characteristic equation.

Physically an unstable system whose natural response grows without bound can cause damage to the system, to adjacent property, or to human life.

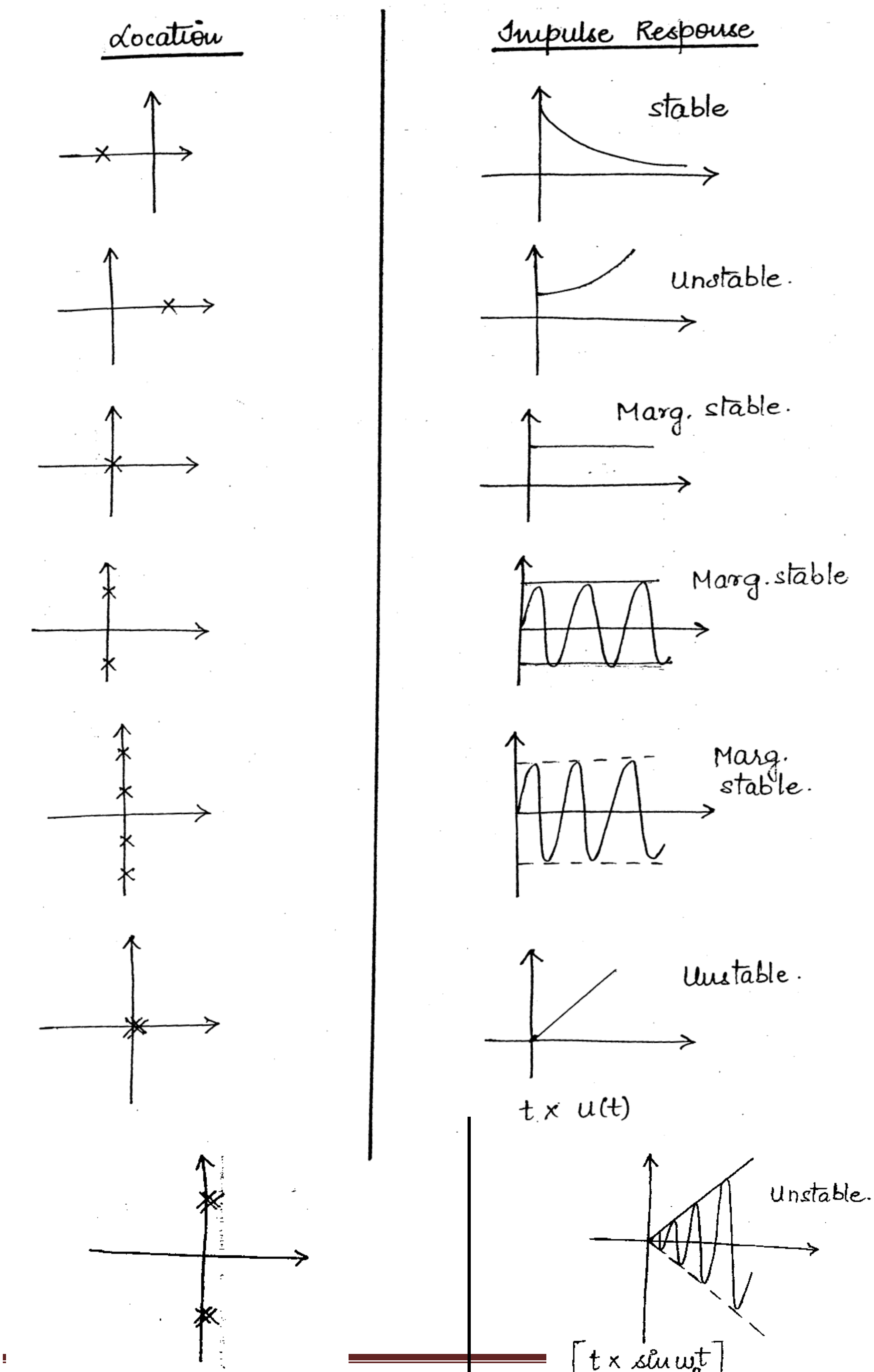
- In the absence of the input, the output tends towards zero irrespective of initial conditions. This stability concept is knows as ‘asymptotic stability’
- If a system output is stale for all variations of the parameters, then the system is called ‘absolutely stable system’. It is quantative measure of stability.
- If a system output os stable for a limited range of variation of its parameters then the system is called ‘conditionally stable system’.
- Relative stability is a quantitative measure of stability. It is a qualitative measure of how fast the transients die out in the system.
- A linear time invariant system is marginally stable if the natural response neither decay nor grows but remains constant or oscillates as time approaches infinity.

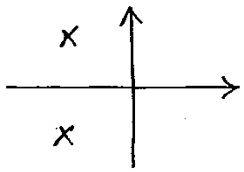
BIBO STABILITY :

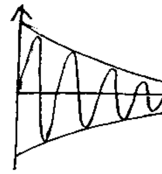
- $\int_0^\infty g(t)\, dt$ is finite i.e. area under the absolute value curve of the imulse response g(t) is evaluated from t = 0 to t = ∞ must be finite.

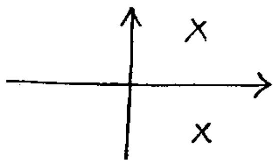
- The nature of impulse response g(t) is dependent on the poles of the transfer functin G(s) which are the roots of the characteristic equation.

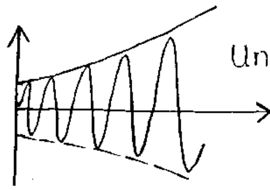
RESPONSE TERMS CONTRIBUTED BY VARIOUS TYPES ROOTS :











Summary

- If all the roots of characteristic equation have negative real parts the system is stable.
- If any root of the characteristic equation has a positive real part (or) if there is a repeated root on the $j\omega$ axis, the system is unstable.
- If all the roots of characteristic equation have negative real parts except for the presence of one (or) more non-repeated roots on the $j\omega$ axis, the system is marginally (or) limitedly stable.

ROUTH-HURWITZ (RH) STABILITY CRITERIA:

Necessary conditions for stability

$q(s) = a_0s^n + a_1s^{n-1} + \ldots + a_n = 0$, $a_0 > 0$

- All the coefficients of its characteristic equation be real and have same sign.
- None of the coefficients should be zero.

Purpose
- To find the closed loop system stability
- To find the number of poles lies left (or) right (or) imaginary axis of the s-plane
- To find the range of k values for stability
- To find the 'k' value for which the system becomes marginally stable.
- To find the frequency of oscillations, when the system is marginally stable.
- To find the relative stability, by using relative concept we can find time constant and settling time and time required to reach steady state.

Note :.
In RH criteria to find the closed loop system stability we require characteristic equation, where as in remaining stability methods require a open loop transfer function.
The characteristic equation of the system is, $q(s) = a_0s^n + a_1s^{n-1} + \ldots + a_n = 0$, $a_0 > 0$

ROUTH ARRAY

s^n	a_0	a_2	a_4	a_6

s^{n-1}	a_1	a_3	a_5	a_7
s^{n-2}	b_1	b_2	b_3	b_4
s^{n-3}	c_1	c_2	c_3	
s^0	a_n			

where $b_1 = \dfrac{a_1.a_2 - a_0 a_3}{a_1}$

$b_2 = \dfrac{a_1.a_4 - a_0 a_5}{a_1}$

- This process continues until s^0 is obtained, which is equal to a_n.
- All the terms in the first column of RH array should have same sign and there should not be any change of sign. This is necessary and sufficient condition for the system to be stable. Any change of sign in the first column of Routh's array indicates,
  - (i) The system is unstable
  - (ii) The number of changes of sign gives the number of roots lying in the right hand side of s-plane.

Difficulty 1 :

The first element in row of Routh array is zero while the rest of the row has atleast one non zero element.

Remedy :

- Replace the first zero element by a small positive number $\in$ and proceed with the formulation of the rest of the RH array. Then examine the signs of the elements in the first column of the completed Routh array as $\in \rightarrow 0$.
- The number of sign changes in the elements of first column of Routh array indicates the number of the characteristic equation in the right hand side of s-plane.

Difficulty 2 :

The elements of one row of Routh tabulation are all zero.

Remedy :

- From the auxiliary equation A(S) = 0 by use of the coefficients form the row just proceeding the row of zeros. Auxilary equation always even polynomial.
- Take derivative of the auxiliary equation with respect to 's' this gives $\dfrac{dA(S)}{dS} = 0$

- Replace the row of zeros with the coefficients of $\frac{dA(S)}{dS} = 0$

Note :
Above special case indicates that one (or) more of the following conditions exist.
1. The equation has atleast one pair of real roots with equal magnitude but opposite sign.
2. The equation has one (or) more pairs of imaginary roots.
3. The equation has pairs of complex conjugate roots forming symmetry about the origin of the s-plane

Relative Stability Analysis :

We require to know the settling time of the dominant roots so as to calculate the relative stability.
- It is inversely proportional to the real part of roots.
- The characteristic equation is modified by shifting the origin of s-plane to S = $-\sigma_1$, by substituting, S = Z - σ_1. Now if new equation satisfies Routh criterion then all roots original equation are more negative than $-\sigma_1$.

ADVANTAGES OF ROUTH'S CRITERION :

i. Stability of the system can be judged without actually solving the characteristic equatin.
ii. No evaluation of determinants, which saves calculation time.
iii. For unstable system it gives number of roots of characteristic equation having positive real part (lies rightside of the S-plane).
iv. Relative stability of the system can be easily judged.
v. By using this criterion, critical value of system gain can be determined hence frequency of sustained oscillations can be determined.
vi. It helps in finding out range of values of K for system stability (conditional stability).
vii. It helps in finding out intersection points of root locus with imaginary axis.

LIMITATIONS OF ROUTH'S CRITERION :

i. It is valid only for real coefficients of the characteristic equation.
ii. It does not provide exact locations of the closed, poles in left or right half of s-plane.
iii. It does not suggest methods of stabilizing an unstable system.
iv. Applicable only to linear systems.
v. RH criterion is a simple statistical technique that determines closed loop stability in limited manner.
vi. The actual roots causing unstability cannot be determined.
vii. Relative stability analysis is not possible.
viii. It is not possible to form Routh-Array if the coefficients of characteristic equation is complex or if the TF consists of infinite sequences.
ix. RH criterion determines closed loop stability from closed loop transfer function unlike other techniques that determine closed loop stability from just open loop Transfer function.
x. It is applicable only if the characterstic equation consisting of finite number of terms.
xi. It is not applicable if the characterstic equation consisting sine, cosine and exponential terms but there is an approximate solution for exponential term.

NOTE : The condition for sustained oscillations is obtained from s^1 row and the frequency of oscillations is obtained from s^2 row of Routh-Array.

ROOT LOCUS DIAGRAM

Root Locus describes qualitatively the changes in the transient response and stability of a system as a system parameter is varied.

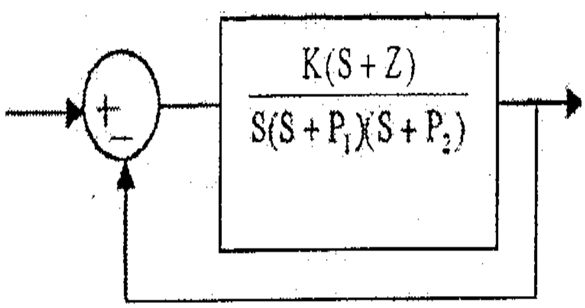
Purpose :

1. To find the CLCS stability
2. To find the range of ‘K’ value for system stability
3. To find the K value to become the system marginally stable
4. To find the un-damped natural frequency of oscillations
5. To find the ‘K’ value for un-damped, under damped, critically damped and over damped systems.
6. To find the relative stability, if the root locus branches moving towards the right then the system is less relatively stable.
7. If the root locus branches moving towards the left the system is more relatively stable..
8. The best method to find the relative stability is root locus.
9. The best method to find the absolute stability is RH-criteria.

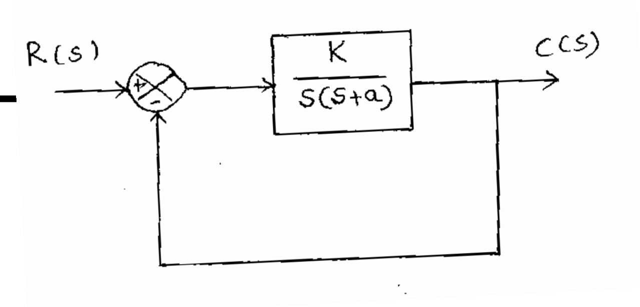
ROOT LOCUS DIAGRAM (RLD):

Root locus diagram introduced by W.R.Evan.

- RLD is a plot of loci of roots of the characteristic equation while gain ‘k’ is varied from 0 to ∞.
- If gain ‘K’ is varied from - ∞ to 0 the diagram is called as inverse or complimentary RLD.
- If more than one parameter is varied the corresponding diagram is called as root contour diagram.



Where K = gain of the system



THE ROOT LOCUS CONCEPT :

The open loop transfer function of the system is

$$G(s) = \frac{k}{s(s+a)} \quad \ldots\ldots\ldots\ldots..(1)$$

Where 'k' and 'a' are constants.

The closed loop transfer function of the system with unity feed back is given by

$$\frac{C(S)}{R(S)} = \frac{k}{s^2 + as + k} \quad \ldots\ldots\ldots.(2)$$

From equation (2) the characteristic equation of the system is,

$s^2 + as + k = 0 \quad \ldots\ldots\ldots\ldots..(3)$

From above equations we can say the closed loop system poles depends on 'k' value

ROOT LOCUS

It is defined as the locus of the closed loop poles as a function of open loop gain (k), when k is varied from 0 to ∞

INVERSE ROOT LOCUS

It is defined as the locus of the closed loop poles as a function of open loop gain (k) is when k is varied from $-\infty$ to 0

ANGLE AND MAGNITUDE CONDITIONS

The closed loop transfer function poles can be obtained from, 1 + G(s).H(S) = 0

G(s).H(s) = -1

(or) G(s).H(s) = $1\angle\pm180^0$

and

G(s).H(s) = $\pm(2q+1)\,180^0$

Actually $-1 + j0 = 1\angle\pm180^0$, the point -1 + j0 is a point of the negative real axis and it can be traces as magnitude 1 at an angle $\pm180^0$, $\pm540^0$, $\pm900^0$, ….., $\pm(2q+1)180^0$

Any point on root locus must satisfy either the magnitude or angle condition.

RULES TO CONSTRUCT ROOT LOCUS

1. Root locus is symmetrical about real axis.
2. The number of branches of root locus = number of poles.
3. The branch of the root locus always start from open loop pole for k = 0 and ends either at open loop zero or at infinity as $k \to \infty$.
4. A point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd.
5. The (P-Z) number of branches tends to infinity along asymptotes whose angles are ϕ_A.

$$\phi_A = \frac{(2q+1)180^0}{P-Z} \qquad q = 0, 1, 2 \ldots (P-Z-1)$$

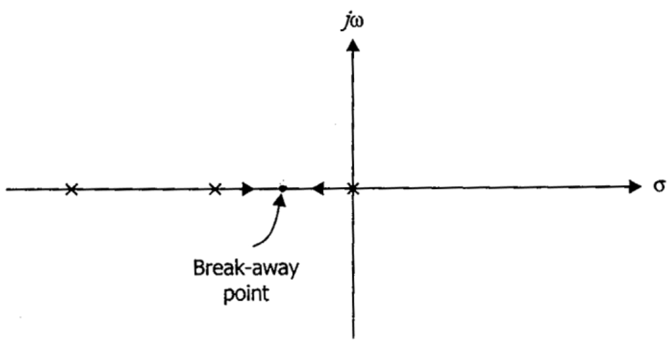
P – Z	ϕ_A
0	-
1	180^0
2	90^0, 270^0
3	60^0, 180^0, 300^0
4	45^0, 135^0, 225^0, 315^0

6. The point of intersection of asymptotes with the real axis known as centroid is given by σ_A.

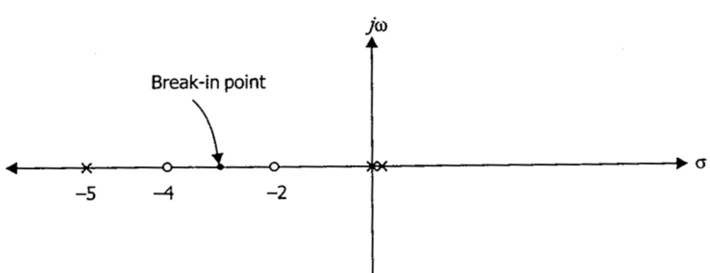
$$\sigma_A = \frac{sum\ of\ \mathrm{Re}[P] - sum\ of\ \mathrm{Re}[Z]}{P - Z}$$

7. **BREAK-AWAY / BREAK-IN POINT**: Break-away point is defined as the point at which root locus comes out of the real axis and break-in point is defined as a point at which root locus enters the real axis. The break-away or break-in points are the points on the root locus at which multiple roots of the characteristic equation occur. If a is the number of branches, the locus leaves the root locus at an angle of $\pm\frac{180^0}{a}$. The general predictions for existence of break-away or break-in points are given below.

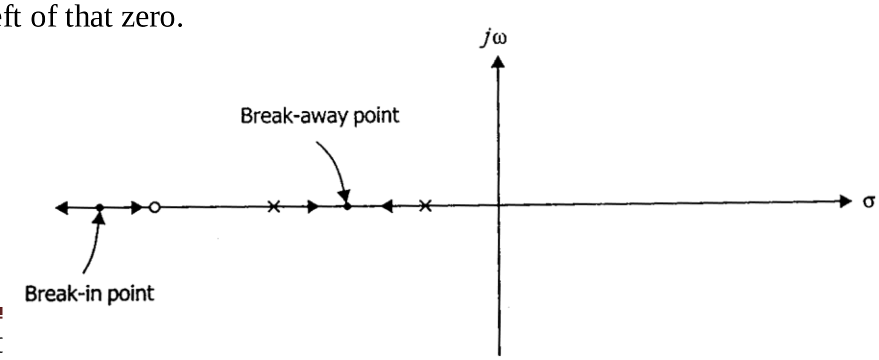
Prediction 1: If there are two adjacently placed poles on the real axis and the real axis is a part of the root locus, one minimum break-away point exists in between adjacently placed poles as shown in Fig.



Prediction 2: If there are two adjacently placed zeros on the real axis and the real axis is a part of the root locus, one minimum break-away point exists in between adjacently placed zeros.



Prediction 3: If there are no poles and zeros to the left of a zero on the real axis and this portion of the real axis is a part of the root locus, there exists minimum one break-in point to the left of that zero.



STEPS TO DETERMINE THE BREAK-AWAY POINTS

Step 1: Frame the characteristic equation 1 + G(s)H(s) = 0 of the system.

Step 2: Write k in terms of s, i.e., k = f(s)

Step 3: Derive $\frac{dK}{ds}$ and put $\frac{dK}{ds}$ = 0

Step 4: The roots of the equation $\frac{dK}{ds}$ = 0 are the break-away points

If the value of K is positive for any root of $\frac{dK}{ds}$ = 0, the root (s) is (are) valid break-away/break-in point (s).

8. Intersection of root locus with jω-axis: To find the intersection of root locus with the imaginary axis, the following procedures are followed.

Step 1: Construct the characteristic equation 1 + G(s)H(s) = 0

Step 2: Develop Routh's array in terms of K.

Step 3: Find K_{mar} that creates one of the roots of Routh's array as a row of zeros.

Step 4: Frame auxiliary equation A(s) = 0 with the help of the coefficient of a row just above the row of zeros.

Step 5: The roots of the auxiliary equation A(s) = 0 for K = K_{mar} give the intersection points of the root locus with the imaginary axis.

9. **Angle f departure/arrival** : The root locus leaves from a complex pole and arrives at a complex zero. These two angles are known as angle of departure and angle of arrival, respectively. Angle of departure (θ_d) is given by

$$\theta_d = 180^0 - \varphi \qquad \text{where } \varphi = (\Sigma\varphi_p - \Sigma\varphi_z)$$

where φ is the angle of G(s)H(s) excluding the pole where the angle is to be calculated.
It s also possible to calculate φ graphically. Similarly, the angle of arrival is given by

$$\theta_a = 180^0 + \varphi$$

Where φ is excluding the zero where the angle is to be calculated.

STEPS FOR SOLVING PROBLEMS ON ROOT LOCUS

Step 1: Determine the branch number of loci, ending at infinity using Rule 1.

Step 2: plot the poles and zeros on s-plane.

Step 3: Find real axis loci using Rule 4. Show the real axis loci wherever present by dark lines.

Step 4: Find the number of asymptotes and their angles by Rule 5.

Step 5:Using Rule 6 determine the centre of asymptotes and draw results of Steps 4 and 5.

Step 6: Determine the break-away/break-in point if present using Rule 7 and mark the point only.

Step 7: Calculate the angle of departure or the angle of arrival due to complex poles or zeros, respectively, using Rule 8.

Step 8: Determine jω crossover using Rule 9 if the locus crosses the jω axis.

GAIN MARGIN FROM ROOT LOCUS

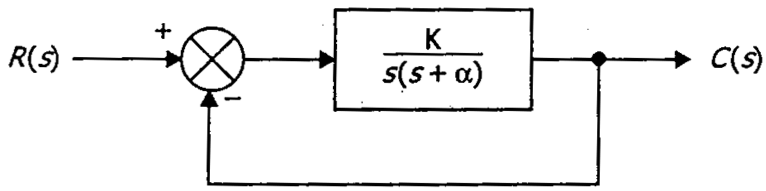
The ratio of 'K' at imaginary crossover to the design value of 'K' of the system is called the value of gain maximum for the rot locus of the system.

$$GM = \frac{Value\ of\ \ K\ at\ imaginary\ crssover}{Design\ value\ of\ 'K'}$$

Note: If the root locus does not cross the jω axis then given is ∞

ROOT CONTOUR

Figure shows a closed – loop system.



Closed loop system

For the above system the study of stability via the root locus technique can be extended by varying any variable other than K from 0 to ∞.

Here

$$G(s)\ H(s) = \frac{K}{s(s+\alpha)}$$

In equation the parameter other than K (e.g. α) can be varied from 0 to ∞ to get the locus of the characteristic equation which is called root contour of the system. During sketching the root contours, the parameters, e.g., α, K, are to be varied simultaneously.

EFFECTS OF ADDITION OF POLES

The effects of addition of poles are as follows:

- There is change in shape of the root locus and it shifts towards the imaginary axis.
- The intercept on the jω axis occurs for a lower value of K because of asymptote's angle being lower down.
- System becomes oscillatory.
- Gain margin and relative stability decrease.
- There is reduction in the range of K.
- A sluggish response can be changed to a quicker response for artful introduction of a pole.
- Settling time increases.

EFFECTS OF ADDITION OF ZEROS

The effects of addition of zeros are as follows:

- There is change in shape of the root locus and it shifts towards the left of the s-plane.
- Stability of the system is enhanced.
- Range of K increases.
- Settling time speeds up.

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IMPORTANT POINTS:

- To evaluate the performance of the system with respect to time is called as time domain analysis.
- Transient state analysis is the part of the time response which deals with the actual (or) natural response of the system after applying the input.
- Steady state analysis is a part of the time response which deals with the calculation of error. The error at $t \to \infty$ is called as steady state error.
- Steady state error cannot be calculated for impulse response as there is no reference level.
- Unit step response of first order system,

  $$C(t) = (1 - e^{-t/T})\, u(t)$$
- The time taken to reach 63% of the final output is called the time constant.
- The transient state analysis depends upon the system time constant where as steady state analysis depends; upon the reference input.
- The system which is having less time constant is faster than the system which is having more time constant, so the speed of the system will be increased as we move away from the origin in the left hand side of the s-plane.
- If we apply sinusoidal input to the linear time invariant (LTI) system the output is also sinusoidal with change in magnitude and phase.
- If the initial value is 'A', final value is 'Z' and T is the time constant of the system then the response of the system with respect to time t is

  $$C(t) = Z + (A - Z)\, e^{-t/T}$$

- Steady state error is only calculated if there is a reference level.
- The steady state error depends upon the input.
- The steady state errors are calculated only for closed loop unity feedback stable systems.
- The steady state error depends on the type of the system.

e_{ss}	Type 0	Type 1	Type 2
Unit step	$\frac{1}{1+K_P}$	0	0
Unit ramp	∞	$1/K_v$	0
Unit parabolic	∞	∞	$\frac{1}{K_a}$

- As the type of the system increases the steady state error decreases, to increase the type of the system we have to integrate the system but it makes the system unstable.
- Standard second order system transfer function;

$$\frac{C(S)}{R(S)} = \frac{w_n^2}{S^2 + 2\xi w_n S + w_n^2}$$

Sl. No.	Range of ξ	Types of closed loop poles	Nature of response	System classification
1	$1 < \xi < \infty$	Real, unequal & negative	Purely exponential	Over damped
2	$\xi = 1$	Real, equal & negative	Critically pure exponential	Critically damped
3	$0 < \xi < 1$	Complex conjugate with negative real part.	Damped oscillations	Under damped
4	$\xi = 0$	Purely imaginary	Oscillations with constant frequency and amplitude	Un-damped

- If ξ increases then T decreases
  t_s decreases
  ω_n decreases
  ω_d decreases
  t_r, t_p, t_d increases
  % M_p decreases
  e_{ss} decreases
  stability increases
  θ decreases
  $\cos\theta$ increases.

- A linear time-invariant system is called to be stable, if the output eventually comes back to its equilibrium.

- A linear time invariant system is called as unstable if the output continues to oscillate or increases unboundly from equilibrium state under the influence of disturbance.
- If the impulse response of a system is absolutely integrable,.

  i.e., $\int_0^\infty |h(t)\, dt| < \infty$ then the system is said to be stable.
- Following conclusions can be drawn:
  - If roots have negative real part $\rightarrow$ impulse response is bounded. System stable.
  - If roots have positive real part system unstable.
  - If roots are repeated (more than 2) on imaginary axis $\rightarrow$ system is unstable.
  - If roots are simple but non repeated (one or more) on imaginary axis $\rightarrow$ system is unstable.

  x(t) is bounded by $\int x(t)\, dt$ is not finite; output is oscillatory.
- Closed loop poles in the right half s-plane are not permissible as the system becomes unstable.
- Roots have negative real part and also one or more non repeated roots on $j\omega$ axis then system is limitedly stable.
- The necessary and sufficient condition for system to be stable is "All the terms in the first column of Routh's array must have same sign. There should not be any sign change in first column of array." If there are any sign changes existing then.
  a) System is unstable
  b) The number of sign changes equals the number of roots lying in the right half of the s-plane.
- Routh's Criterion is valid only for real coefficients of the characteristic equation.

- To find the relative stability, if the root locus branches moving towards the right then the system is less relatively stable.
- If the root locus branches moving towards the left the system is more relatively stable.
- The best method to find the relative stability is root locus.
- The best method to find the absolute stability is RH-criteria.
- Root locus means closed loop poles path by varying 'K' from 0 to ∞.

  K $\rightarrow 0\ to\ \infty \rightarrow$ Root locus diagram

  K $\rightarrow 0\ to -\infty \rightarrow$ complementary root locus diagram

  K $\rightarrow -\infty\ to\ \infty \rightarrow$ complete root locus diagram

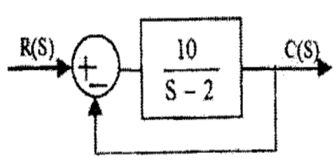
  K_1, K_2 $\rightarrow 0\ to\ \infty \rightarrow$ Root contour diagram
- The closed loop poles are nothing but the sum of open loop poles and open loop zeros with the function of system gain 'K'
- When K = 0, closed loop poles = open loop poles
- When K = ∞, closed loop poles = open loop zeros
- The root locus diagram starts at open loop pole where K = 0 and ends at open loop zero where K = ∞
- To draw a root locus diagram the number of poles must be equal to number of zeros. If zeros are less then we will assume zeros at ∞. The direction of ∞ given by the angle of asymptotes.
- Centroid is always real, it may be located on negative real axis. It may or may not be part of root locus.
- If there are adjacent placed poles on the real axis and the real axis between them is a part of the root locus then there exists minimum and breakaway point in between adjacently placed poles.

- If there are two adjacently placed poles or zeros on real axis and section of real axis in between them is a part of root locus then there exists minimum one breakaway point in between adjacently placed poles or zeros.
- If there is a zero on the real axis and to the left of than zero there is no pole or zero existing on the real axis and complete real axis to the left of this zero is a part of the root locus then there exists minimum one breakaway point to the left of them zero.
- Root locus is the technique employed to find roots of characteristic equation. This technique provides a graphical method of plotting the locus of roots. It brings into focus the dynamic response of the system.
- If any point in s-plane has to be on root locus then it has to satisfy $\angle G(s)$ H(s) for any value of 's' which is root of the equation 1 + G(s) H(s) = 0 = ± (2q + 1) 180^0, |q = 0, 1, 2, 3…..
- Gain margin = $\frac{value\ of\ \ k\ at\ the\ stability\ boundary}{design\ value\ of\ \ k}$

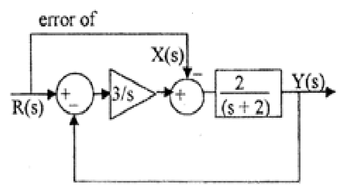
PRACTICE QUESTIONS:

1. Step response of a system is $10(1-e^{-t})$ u(t) . Find the TF.

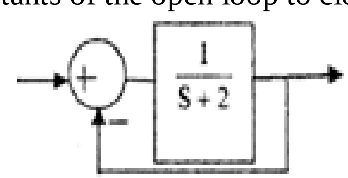
2. For the system shown below find the initial and final values to a step input.



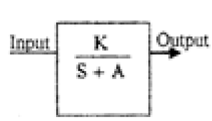
3. $X(S) = \dfrac{10(s+4)}{(s+1)(s+2)}$, ii) $X(S) = \dfrac{10(s+1)}{(s+10)}$
   find the initial & final values of the above function X(t),
   Where $X(t) = L^{-1}[X(S)]$
4. $TF = \dfrac{10(s+1)}{(s+10)}$ find the initial & final value to a step input
5. $F(s) = \dfrac{1}{(s^2+1)}$ Find the final value of f(t) is
6. $TF = \dfrac{10(s+10)}{(s+5)}$ find the initial & final value to an Impulse input.
7. When subjected to a unit step input, the closed loop control system shown in the figure, find the steady state error

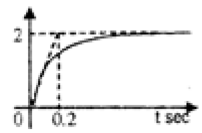


8. $TF = \dfrac{100}{(s+10)(s^2+2s+2)}$ find the dominant poles of the 2nd order approximation of the above TF
9. The transfer function of a system is $G(s) = \dfrac{100}{(S+1)(S+100)}$. For a unit –step input to the system, find the approximate settling time for 2% criterion
10. find the ratio of time constants of the open loop to closed loop TF of the following system is

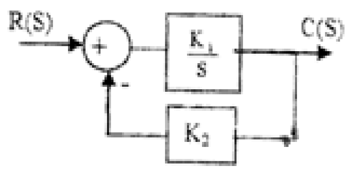


11. TF of a system is $\dfrac{A}{S+A}$ and it takes 6sec to reach 95% of the response to a step input . Find the value of ‘A’ is
12. System and its step response is given below, find ‘K’ & ‘A’

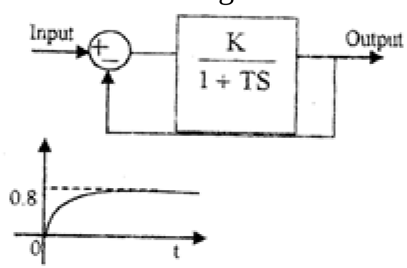




13. The steady state gain is 2 & time constant is 4 sec, find the values of k_1 & k_2 are respectively



14. The system and its step response is shown in fig below. Find the value of 'K' is



15. A control system with certain excitation is governed by the following mathematical equation $\frac{d^2x}{dt^2} + \frac{1dx}{2dt} + \frac{1}{18}\ x = 10 + 5e^{-4t} + 2e^{-5t}$ Find the natural time constants of the response of the system

16. A linear , time – invariant , casual continuous time system has a rational transfer function with simple poles at S= -2 and S = - 4, and one simple zero at S= -1.A unit step U(t) is applied at the input of the system . At steady state, the output has constant value of 1 . Find the impulse response of this system.

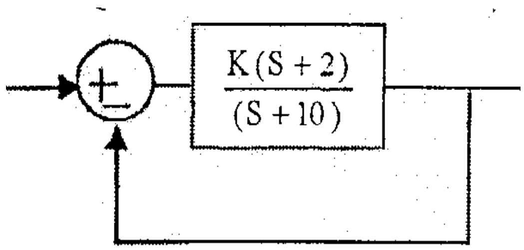
17. TF of a system is $\frac{10(S+5)}{S^3+6S^2+11S+6}$ test whether the system is stable (or) not.

18. TF= $\frac{5(S+4)}{2S^4+S^3+3S^2+5S+10}$ find the no. of left hand, right hand and $j\omega$ axis poles of the above system.

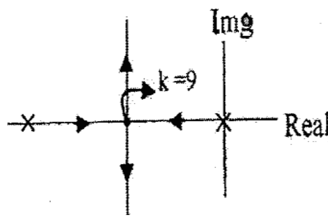
19. G(S)H(S)= $\frac{4}{S^2}$ test the stability of a system.

20. CE is $S^3 - 4s^2 - 5S + 6 = 0$ find the no of roots present in the right side of s- plane.

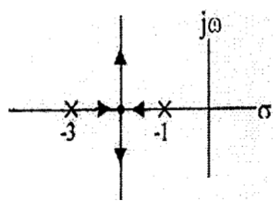
21. Find the angle of asymptotes and the Centroid for the system given to the following system.



22. The RLD of a certain system is given below , then find the OLTF G(S) H(S)



23. Given the root locus of a system G(S) = $\frac{4k}{(S+1)(S+3)}$



What will be the gain for obtaining the damping ratio 0.707

24. G(S) H(S) = $\frac{K(S+3)}{S(S+2)}$

i) Find the break in or break away points and sketch the RLD

ii) Find the value of ‘k’ at S = - 1 and – 4

25. G(S) H(S) = $\frac{K(S)(S+2)}{(S+4)(S+6)}$ Find break away points on the RLD

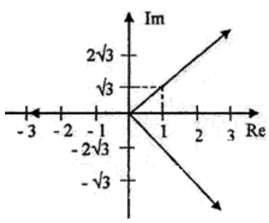
26. Characteristic equation of a system is $S^4 + 3S^3 + 7S^2 + (3 + k) S + 3K = 0$, then find the angle of asymptotes

27. G(S) H(S) = $\frac{K}{S(S+1)(S+2)}$

Find the point of intersection of the RLD w.r.to imaginary axis and also find the value of K at that point

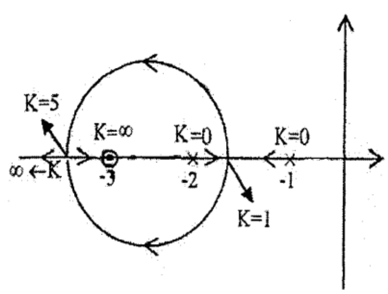
Root Locus plot intersects the jω axis

28. Figure shows the root locus plot (Location of poles not given) of a third order system, then find the open Loop transfer function



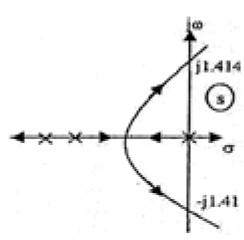
29. Check whether the points s_1 = - 3 + j4 and s_2 = - 3-j2 in the s – plane are part root locus for the system with the open – loop transfer function G(s) H(s) = $\frac{K}{(s+1)^4}$

30. The root – locus diagram for a closed loop feed back system is shown in figure. The range of K such that system is over damped over



31. The Routh – Hurwitz array is given for a third order characteristic equation

S^3	1	b	0	0
S^2	a	c	0	0
S^1	$\frac{(6-k)}{3}$	0		
S^0	k			



The coefficient a, b, c and k are such that a = 3, b > 0, k > 0, c > 0 and c is a function of k. The root Locus for the corresponding characteristic equation is as shown in the given figure, then find the values of k and C for critical stability

32. Consider the second – order system with the characteristic equation s(s + 3) + K(s + 5) = 0 based on the properties of the root loci, it can be shown the complex portion of the root loci of the given System for $0 < K < \infty$ is described by a circle, and find the two breakaway points on the real axis

33. Root locus of s(s + 2) + K(s + 4) = 0 is a circle. What is the co – ordinates of the centre of this Circle?

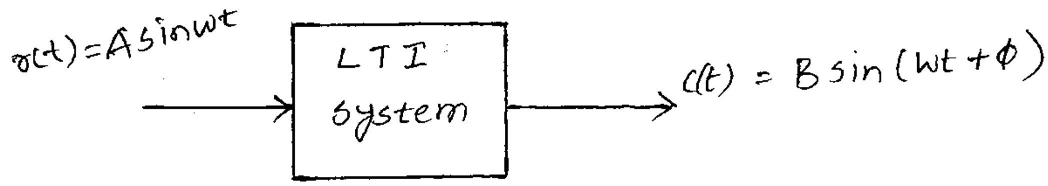
34. Find the gain at the breakaway point of the root locus along the real axis of a unity feedback system with the transfer function G(s) = $\frac{K}{s^2+s-1}$

35. The loop transfer function of a unity feedback system is given by G(s) = $\frac{(s-1)(s-K)}{(s+1)(s+4)}$. For $0 \le K \le \infty$ find its root locus starting points

UNIT – IV

FREQUENCY RESPONSE ANALYSIS

- The magnitude and phase relationship between sinusoidal input and steady state output is frequency response



Where A = Amplitude of the system

ϕ = phase of the system

A linear system with a sinusoidal input r(t) = ASin ωt under steady state the system output as well as signals all other points in the system are sinusoidal. The steady state output may be written as c(t) = BSin (ωt+φ).

The magnitude and phase relationship between the sinusoidal input and the steady state output of a system is term as the frequency response.

Resonance frequency: The frequency at which peak resonance M_r occurs is known as resonance frequency (ω_r).

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

Bandwidth (BW): The frequency at which the magnitude of M(jω) drops by -3 dB below a certain specified level, generally 0 dB level. If bandwidth is large, higher frequencies will pass through the system. This indicates a faster rise in time. Therefore, bandwidth gives an idea about transient response. Again, small bandwidth means lower frequencies are passed and hence response is sluggish. From bandwidth, noise filtering can be known because large bandwidth indicates susceptibility to noise. Bandwidth also implies the sensitivity to parameter variations.

$$BW = \omega_n \sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}$$

Gain Margin (GM):

It is defined as the amount of additional gain that should be added at the phase crossover frequency to bring the system to the verge of instability.

- Phase crossover frequency is the frequency at which the phase angle of the transfer function is -180^0.
- It is a measure of relative stability.

Phase Margin (PM):

Φ_{pm} is defined as the amount of additional phase that should added at gain crossover frequency to bring the system to the verge of instability.

- Gain crossover frequency is the frequency at which the gain is unity.
- It is a measure of relative stability.

Time Response		Frequency Response	
1	$\omega_d = \omega_n \sqrt{1-\xi^2}$	1	$\omega_r = \omega_n \sqrt{1-2\xi^2}$
2	$M_p = e^{-\xi\pi} / \sqrt{1-\xi^2}$	2	$M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$
3	$\phi = \tan^{-1}\left(\dfrac{\sqrt{1-\xi^2}}{\xi}\right)$	3	$\phi_r = -\tan^{-1}\left(\dfrac{\sqrt{1-2\xi^2}}{\xi}\right)$
4	$t_r \alpha \dfrac{1}{speed}$	4	$B.W \ \alpha \ speed$
5	$t_s \alpha \dfrac{1}{stability}$	5	$\left.\begin{matrix} GM \\ PM \end{matrix}\right\} \alpha \ stability$
6	$0 < \xi < 1$	6	$0 < \xi < \dfrac{1}{\sqrt{2}}$

Note: For satisfactory performance, the phase margin should be between 30^0 and 60^0 and gain margin should be greater than 6 dB.

- If a zero added in the forward path of a second order system,
  - Rise time decreases due to increase in bandwidth
  - Settling time will increase
  - System will be more stable
- If a pole added in the forward path of a second order system,
  - Rise time increases due to decrease in bandwidth
  - Peak overshoot increases
  - System will be less stable

For satisfactory operation the range of ξ is generally $0.4 < \xi < 0.707$

ADVANTAGES OF FREQUENCY DOMAIN ANALYSIS

- In laboratory, it is possible to obtain a frequency response test with god accuracy and it is very useful when it is very difficult t obtain transfer function by an analytical technique.
- It is very easy to design open-loop transfer function for specified closed-loop performance in frequency domain compared to time domain.
- In frequency domain it is very easy to visualize the effects of noise disturbance and parameter variations.

DISADVANTAGES OF FREQUENCY DOMAIN ANALYSIS

- These methods are basically applicable only to linear systems.
- It is possible to obtain frequency response for an existing system if the time constants are up to few minutes.
- These are time-consuming processes.
- These methods are back dated compared to the extensive methods developed for digital computer simulation and modeling.

FREQUENCY RESPONSE PLOTS

1. Bode plot
2. Polar plot
3. Nyquist plot

BODE PLOT:

- It is also called as asymptotic (or) corner Plot.

Purpose :

1. To draw the frequency response of open loop transfer function.
2. To find the close loop system stability.
3. To find the gain margin, phase margin, gain cross over frequency phase cross over frequency

- Bode plot consists of two plots which are
  1. Magnitude expressed in logarithmic values against logarithmic values of frequency called **Magnitude Plot**.
  2. Phase angle in degrees against logarithmic values of frequency called **Phase Angle Plot**.

Magnitude Plot: $|G(S)H(S)|$ in dB Vs frequency (ω)

Phase Plot: $\angle$G(s) H(s) vs frequency (ω)

Decade and Octave: Change in frequency by a factor of '10' is known as a decade and by a factor of '2' is called as an octave

20dB/decade = 6 dB/octave

.STEPS TO PLOT BODE PLOT:

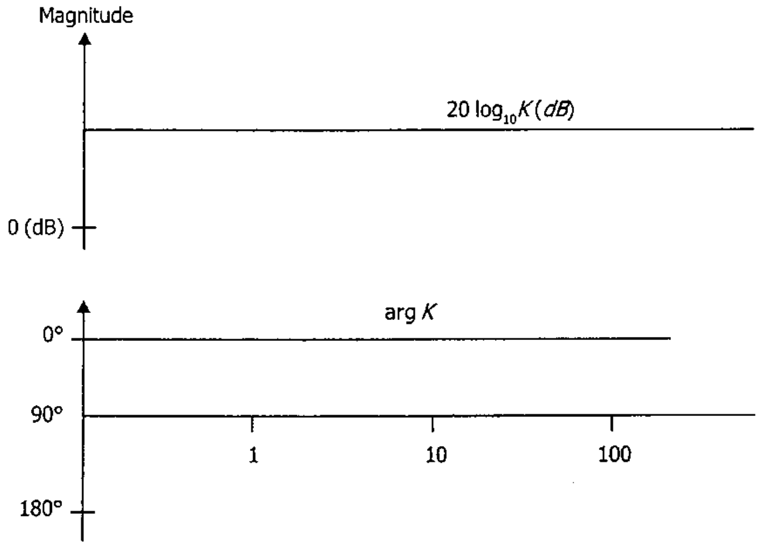
1. Rewrite the sinusoidal transfer function as a product of basic factors in the time constant form.
2. Identify the corner frequencies associated with each one of these basic factors
3. Knowing the corner frequencies, draw the asymptotic magnitude plot. This plot consists of straight line segments with line slope changing at each corner frequency + 20 dB/decade for a zero and -20 dB/decade for a pole ($\pm$20mdB/decade for a zero or pole of multiplicity m). For a complex conjugate zero or pole the slope changes by $\pm$ 40dB/decade.
4. Calculate the total phase angle of G(jω) H(jω) at different frequencies and plot the resultant phase plot.

BODE PLOT FOR THE CONSTANT TERM 'K'

K (in dB) = 20 log|K| = constant

and $\angle K = 0^0$

Figure shows the Bode plot of constant (K).



Bode plot of constant (K).

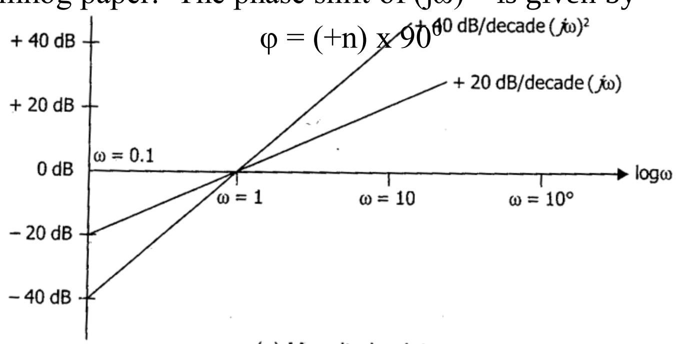
BODE PLOT FOR ZERO AT THE ORIGIN, i.e., $(j\omega)^{+n}$

The magnitude in decibels is given by

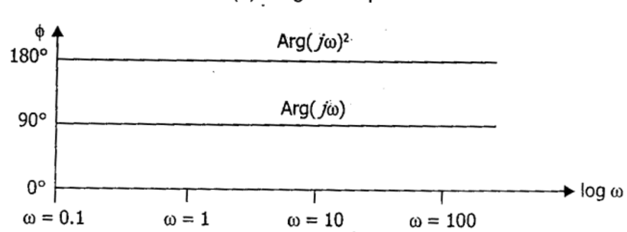
$$20 \log |(j\omega)^{+n}| = +20n \log \omega \text{ (dB)}$$

Equation represents the equation of a straight line having slope of 20n dB/decade or 6n dB/octave which passes through the 0 dB point at ω = 1 in the logarithmic frequency scale on semilog paper. The phase shift of $(j\omega)^{+n}$ is given by

$$\varphi = (+n) \times 90^0$$



(a) Magnitude plot



(b) Phase Plot

BODE PLOT FOR POLES OF THE ORIGIN, i.e., $(j\omega)^{-n}$

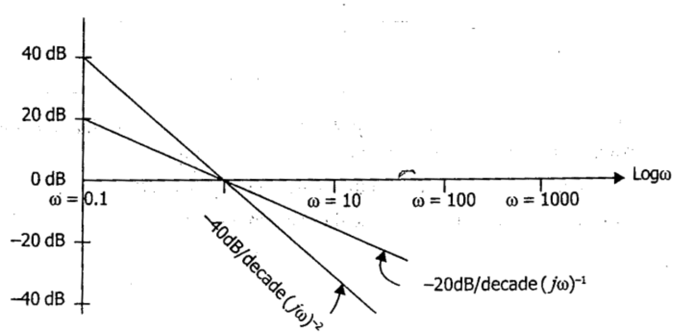
The magnitude in decibels is given by

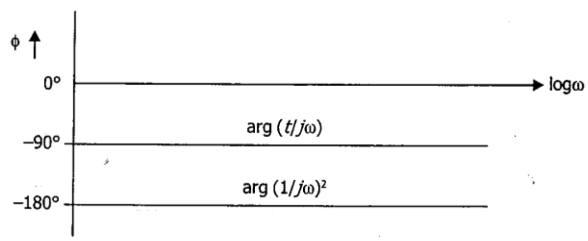
$$20 \log |(j\omega)^{-n}| = -20n \log\omega$$

Equation represents the equation of a straight line having slope of -20n dB/decade or -6n dB/octave and it passes through the 0 dB point at ω = 1 in the logarithmic frequency scale on semilog paper. The phase shift of $(j\omega)^{-n}$ is given by

$$\varphi = (-n) \times 90^0$$

Figure shows the magnitude and phase curves of the term $(j\omega)^{-n}$





BODE PLOT FOR SIMPLE ZERO (1+jωT)

$$G(j\omega)H(j\omega) = 1 + j\omega T$$

Now $\quad 20\log|G(j\omega)\, H(j\omega)| = 20 \log \sqrt{1+\omega^2T^2}$

To plot the magnitude curve, usually a linear asymptotic relation is used. At very low frequencies, ωT << T, equation can be written as

$$20 \log|G(j\omega)\, H(j\omega)| = 20 \log 1 = 0 \text{ dB}$$

At very high frequency, ωT >> 1, equation can be written as

$$20\log|G(j\omega)\, H(j\omega)| = 20 \log \sqrt{\omega^2T^2} = 20 \log\omega T$$

$$20\log|G(j\omega)\, H(j\omega)| = 20 \log \omega + 20 \log T$$

Equation represents a straight line having slope of 20 dB/decade (or 6dB/octave). Corner frequency (ω_c) can be obtained from the intersection of the low-frequency and high-frequency asymptotes as follows:

$$20 \log(\omega_c T) = 20 \log 1$$

or $\qquad \omega_c = \frac{1}{T}$

Now (actual-approximate)

$$= 20 \log \sqrt{(\omega_c T) + (1)^2} \ - 20 \log 1$$

Now at corner frequency,

$$(\text{actual-approximate})|_{\omega=\omega c} = 20 \log \sqrt{1^2 + 1^2} \ - 20 \log 1 = 3.03 \text{ dB}$$

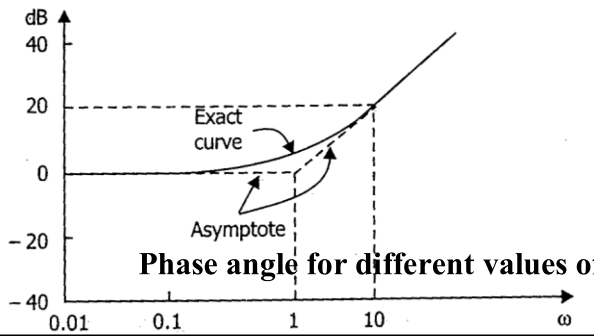
Maximum error occurs at corner frequency. The general error values are given below:

Error for different values of ω

Frequency	$\omega = 0.5\ \omega_c$	$\omega = \omega_c$	$\omega = 2\omega_c$
Error	1 dB up	3 dB up	1 dB up

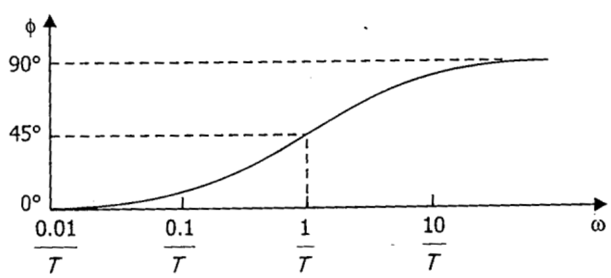
Figure shows the Bode plot of (1+jωT) which shows the actual curve as well as the linear asymptotic curves. The actual or exact plot deviates slightly from straight-line asymptotes.

Now $\angle G$ (jω)H(jω) = $\tan^{-1}$ (ωT). To construct the phase angles plot of G(jω)H(jω) the phase angles of G(jω)H(jω) for different values of ω are calculated and these are given below.



Phase angle for different values of ω

ω	$0.1\ \omega_c$	$0.5\ \omega_c$	$1\ \omega_c$	$2\ \omega_c$	$10\ \omega_c$	∞
$\angle G$ (jω)H(jω)	$+5.71^0$	$+26.6^0$	$+45^0$	$+63.4^0$	$+84.3^0$	$+90^0$



BODE PLOT FOR SIMPLE POLE $(1+j\omega T)^{-1}$

Here G(jω)H(jω) $= (1+j\omega T)^{-1}$

Now 20 log|G(jω) H(jω)| $= -20 \log \sqrt{1 + j\omega^2 T^2}$

At every low frequencies, wT << 1, Equation can be written as

$$20 \log|G(j\omega)H(j\omega)| = -20 \log 1 = 0 \text{ dB}$$

At very high frequencies, ωT >> 1, equation can be written as

$$20 \log|G(j\omega)H(j\omega)| = -20 \log \sqrt{\omega^2 T^2} = -20 \log (\omega T)$$

Now $20 \log|G(j\omega) H(j\omega)| = -20 \log\omega - 20 \log T$

Equation represents a straight line having slope of -20 dB/decade (or 6dB/octave). Corner frequency (ω_c) can be obtained from the intersection of the low-frequency and high-frequency asymptotes as follows:

Crner frequency (ω_c) can be obtained from

$$-20 \log(\omega_c T) = -20 \log 1$$

or $\omega_c T = 1$

or $\omega_c = \frac{1}{T}$

Now (actual-approximate)

$$= -20 \log \sqrt{(\omega_c T) + (1)^2} + 20 \log 1$$

Now at corner frequency,

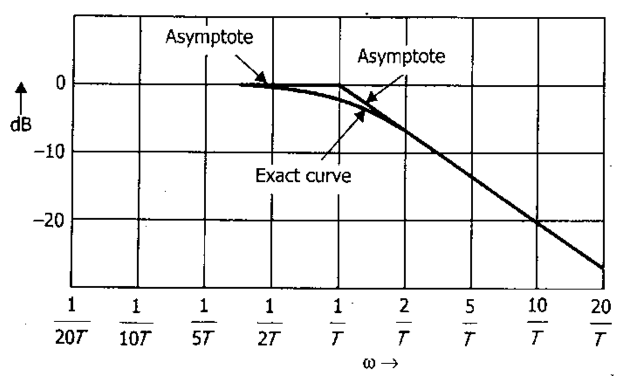
$$(\text{actual - approximate})\big|_{\omega=\omega_c} = -20 \log \sqrt{1^2 + 1^2} + 20 \log 1 = -3.03 \text{ dB}$$

Maximum error occurs at corner frequency. The general error values are given below:

Error for different values of ω

Frequency	$\omega = 0.5\,\omega_c$	$\omega = \omega_c$	$\omega = 2\omega_c$
Error	1 dB down	3 dB down	1 dB down

Figure shows the Bode plot of 1/(1+jωT) which shows the actual curve as well as the linear asymptotic curves. The actual or exact plot deviates slightly from straight –line asymptotes.



The phase shift of G(jω)H(jω) is given by

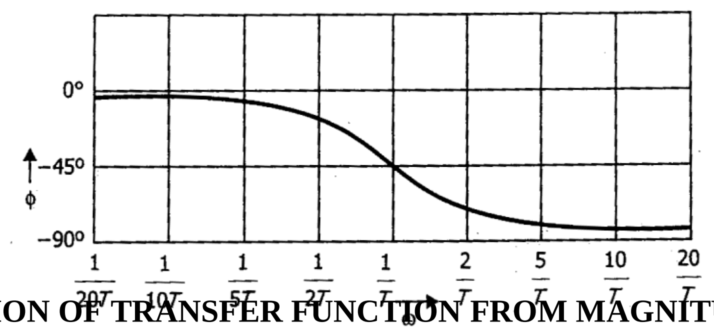
$$\angle[G(j\omega)H(j\omega)] = \varphi = -\tan^{-1}\omega T$$

To construct the phase ange plots of G(jω)H(jω) the phase angle of G(jω) H(jω) for different values of ω are calculated and these are given below.

Phase angle for different values of ω

ω	0.1 ω_c	0.5 ω_c	1 ω_c	2 ω_c	10 ω_c	∞
$\angle G$ (jω)H(jω)	-5.71^0	-26.6^0	-45^0	-63.4^0	-84.3^0	-90^0

Figure shows the phase angle plot of the simple pole 1/1+jωT



CALCULATION OF TRANSFER FUNCTION FROM MAGNITUDE PLOT

1. Identify starting slope. Starting slope of magnitude plot represents poles or zeros at the origin.

   If starting slope - 20 dB/decade there is one pole at origin
   - 40 dB/decade there are two poles at origin
   0 dB/decade there is no pole or zero at origin
   +20 dB/decade there is one zero at origin and so on
2. Identify corner frequencies.
3. The change in slope indicates the respective factor with corresponding corner frequencies.

   If change in slope is -20 dB/decade i.e. -20 to -40 (or) 0 to -20 and so on that the factor is simple pole.

   If change in slope is +20 dB/decade i.e., 20 to 40 or 0 to 20 and so on that the factor is simple zero.
4. Write a transfer function in time constant form (the time constant is reciprocal of corner frequency).
5. Shift the magnitude plot at ω = 1 which represents 20 log K from which we can decide value of 'K'.

STABILITY FROM BODE PLOT:

- **Calculation of gain margin and phase margin from Bode Plot**

  Gain Margin = 0 – [resultant magnitude in dB at ω_{pc}] (or)

  Gain Margin = $-20 \log \left| G(j\omega)\, H(j\omega) \right|_{\omega=\omega_{pc}}$ in dB (or)

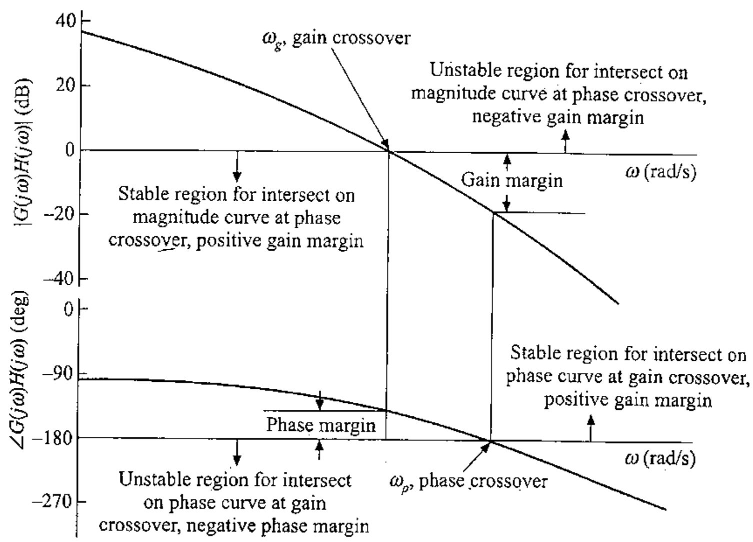
  Gain Margin = $\left. \dfrac{1}{|G(j\omega)H(j\omega)|} \right|_{\omega=\omega_{pc}}$

- If magnitude plot is below the '0'dB line at the ω_{pc}, the gain margin is positive.
- If magnitude plot is above the 0 dB line at the ω_{pc}, the gain margin is negative.

  Phase Margin = $180^0 + \angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{gc}}$
- If phase plot is above the -180^0 line at the ω_{gc} the phase margin is positive.
- If phase plot is below the -180^0 line at the ω_{gc}, the phase margin is negative.

Note:
- If phase margin is positive, gain margin is positive then the system is stable.
- If phase margin is negative, gain margin is negative then the system is unstable.
- If phase margin is zero, gain margin is zero then the system is marginally stable.
- If $\omega_{pc} > \omega_{gc}$ then the system is stable
- If $\omega_{pc} < \omega_{gc}$ then the system is unstable
- If $\omega_{pc} = \omega_{gc}$ then the system is marginally stable

Note:



- Gain Margin and Phase Margins are only defined for open loop systems.

ADVANTAGES OF A BODE PLOT

(i) It is possible to show both the high frequency and the low frequency characteristic of a transfer function in a single diagram.

(ii) Using valid assumptions, the Bode plots can be constructed very easily.

(iii) From a Bode plot it is possible to calculate the Gain Margin and Phase Margin very easily, and hence the relative stability of the system can be studied.

(iv) It is possible to determine the various other frequency specifications such as the cut-off frequency, bandwidth, etc.

(v) From a Bode plot, we get the transfer function of the system.

(vi) Using a Bode plot it is possible to compensate the system to obtain the desired response.

(vii) From a Bode plot it is also possible to design the value of K from the knowledge of Gain Margin or Phase Margin.

(viii) Experimentally, it is possible to draw a Bode plot without the knowledge of the transfer function.

DISADVANTAGES OF A BODE PLOT

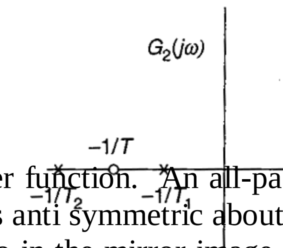
Absolute and relative stability of only minimum phase system can be determined from the Bode Plot.

TYPES OF SYSTEMS:

MINIMUM PHASE-SYSTEM

A minimum-phase system is a system whose transfer function is of minimum-phase type. A minimum-phase transfer function is a transfer function which has got all its poles and zeros only in the left half of the s-plane. Such a transfer function has the least (minimum) phase angle range for a magnitude curve.

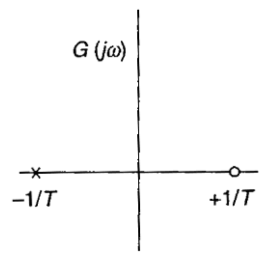
$$G_1(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$



ALL-PASS SYSTEM

An all-pass system is a system with all-pass transfer function. An all-pass transfer function is a transfer function having a pole-zero pattern which is anti symmetric about the imaginary axis (i.e. for every pole in the left half s-plane, there is a zero in the mirror image position with respect to imaginary axis). The magnitude of the all-pass transfer function is unity at all frequencies.

$$G_2(j\omega) = \frac{1-j\omega T}{1+j\omega T}$$



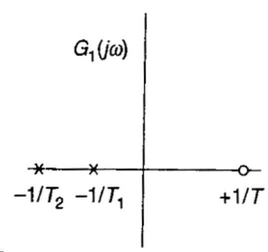
NON-MINIMUM-PHASE SYSTEM

A transfer function which has one or more zeros in the right-half s-plane is known as non-minimum-phase transfer function. A non-minimum-phase transfer function can be treated as a combination of a minimum-phase transfer function and an all-pass transfer function.

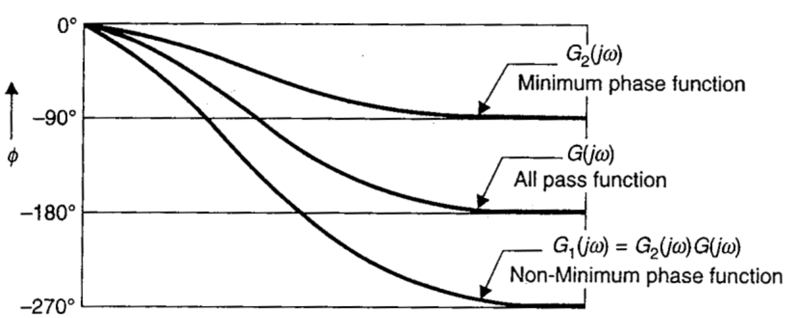
$$G_3(j\omega) = \frac{1-j\omega T}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$G_3(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)} \times \frac{1-j\omega T}{1+j\omega T}$$

$$G_3(j\omega) = G_1(j\omega) \cdot G_2(j\omega)$$



Non-minimum Phase system = Minimum Phase system x All-pall system



A non-minimum phase element is transportation lag which has the transfer function

$$G(j\omega) = e^{-j\omega T} = 1 \angle -\omega T \text{ rad} = 1 \angle -57.3\,\omega T \text{ deg}$$

TRANSPORT LAG

Transport lag is of non-minimum phase behavior has an excessive phase lag with no attenuation at high frequencies. Such transport lag normally exists in thermal, hydraulic and pneumatic systems.

$$G(j\omega) = e^{-j\omega T}$$

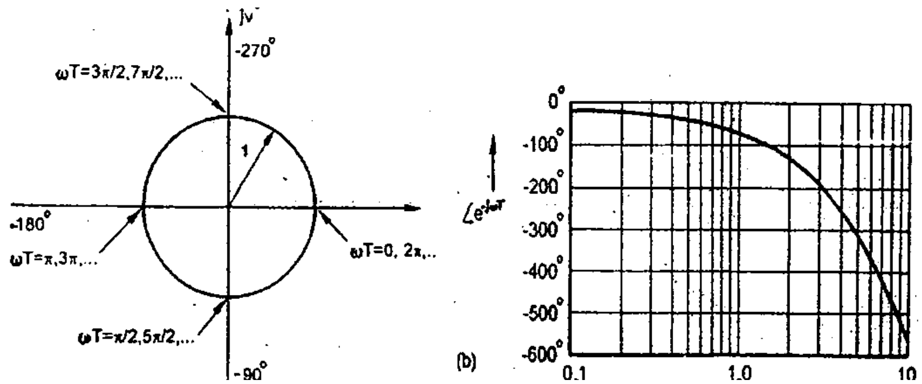
The magnitude is always equal to unity since

$$|G(j\omega)| = |\text{Cos}\,\omega T - j\,\text{Sin}\,\omega T| = 1$$

Therefore the log magnitude of the transport lag $e^{-j\omega T}$ is eaual to 0 dB. The phase angle of transport lag is

$$\angle G(j\omega) = -\omega T \text{ (rad)} = -57.3\,\omega T \text{ (degrees)}$$

The phase angle varies linearly with ω.



POLAR PLOTS

It is a plot of magnitude and phase of G(S) H(S) in polar coordinates while ω is varied from 0 to ∞

- Point of interaction of the polar / Nyquist lot with respect to negative real axis is calculated as given below

Let 'a' be the point of intersection a = $|G(S)H(S)|_{\omega=\omega_{pc}}$

Purpose:

1. To draw the frequency response of open loop Transfer function.
2. To find the closed loop system stability.
3. To find the gain margin & phase margin.
4. To find the relative stability

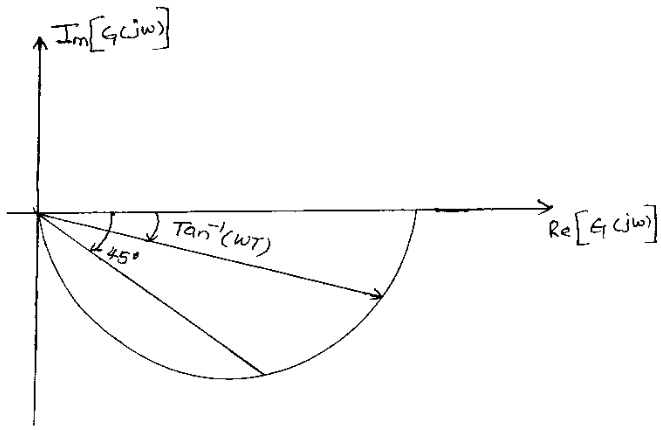
Draw the polar plot for G(S) = $\dfrac{1}{1+ST}$

S = jω

$G(j\omega) = \dfrac{1}{1 + j\omega T} = \dfrac{1}{\sqrt{1+\omega^2 T^2}} \angle -\tan^{-1} \omega T$

$|G(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2 T^2}}$

$\angle G(j\omega) = -\tan^{-1}\omega T$



ω	|G(jω)|	∠G(jω)
0	1	0^0
1/a	0.707	-45^0
10/a	0.1	-84.28^0
100/a	0.01	-89.42^0
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
∞	0	-90^0

PROCEDURE TO DRAW THE POLAR PLOT

Step 1: Find the magnitude & phase at ω = 0 & ω ∞

Step 2: Ending direction = $\phi_\infty - \phi_0$ = positive then anticlockwise direction

= negative then clockwise direction

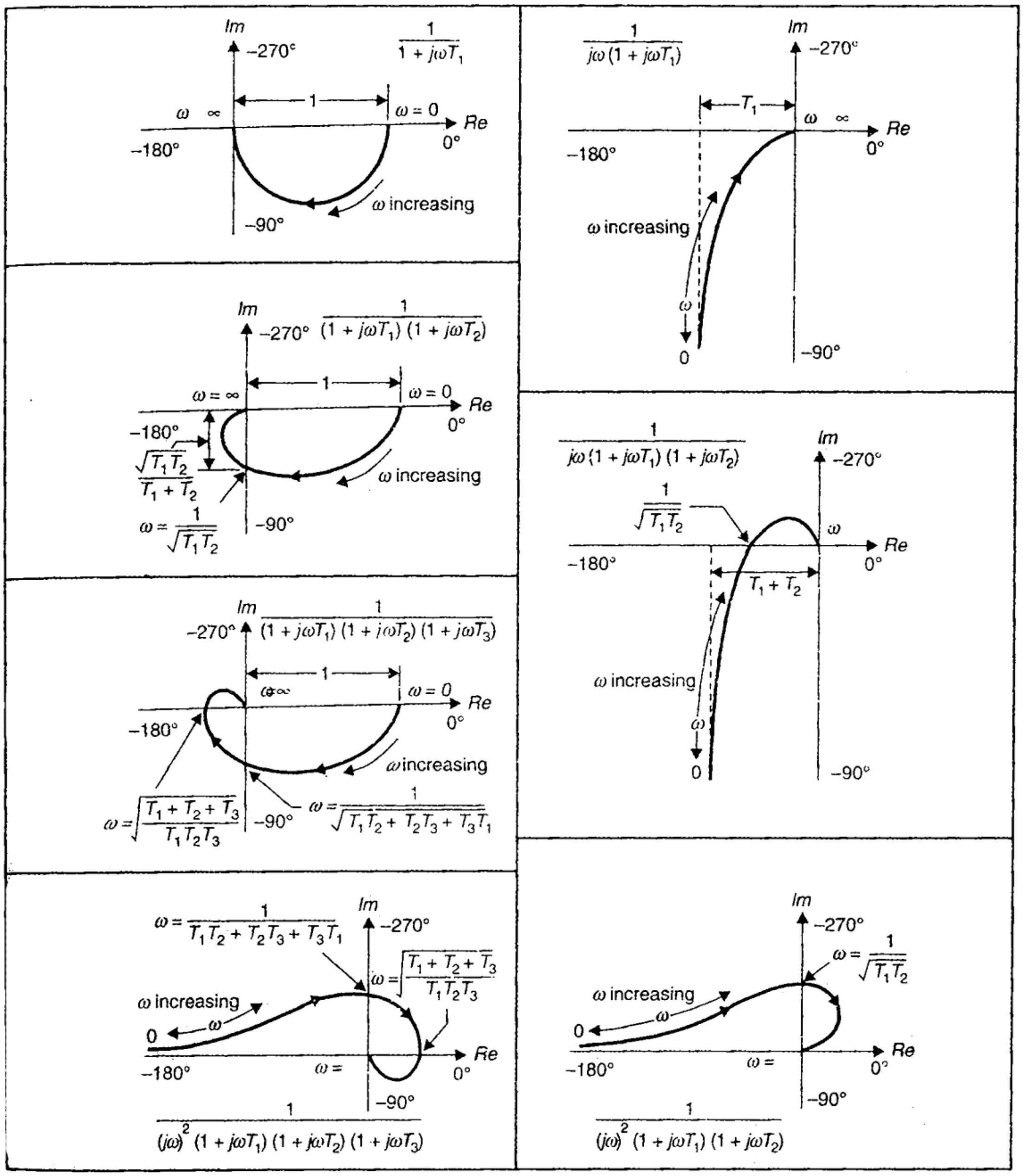
Step 3: Starting direction

The starting direction is considered to the transfer functions it should have only positive sign terms.

- If the finite pole is near to imaginary the starting direction is clockwise
- If the finite zero is near to imaginary the starting direction is anticlockwise

Note:

Above procedure is not valid for when the magnitude at ω = 0 less than the magnitude at ω = ∞ like high-pass filter. In this case get the plot by using standard procedure.



Observations:

(i) Addition of a nonzero pole (finite pole) to a transfer function results in further rotation of the polar plot through an angle of -90^0 as $\omega \rightarrow \infty$.

(ii) Addition of a pole at the origin to a transfer function rotates he polar plot at zero and infinite frequencies by a further angle of -90^0.

(iii) The addition of each finite zero to a transfer function results in further rotation of polar plot through an angle of 90^0 as $\omega \rightarrow \infty$.

(iv) Addition of zero at origin to transfer function results in further rotation of polar plot through an angle of 90^0 as $\omega \rightarrow \infty$ in anticlockwise direction.

NYQUIST PLOT

The Nyquist Plot are developed by using the mathematical principle known as principle of arguments.

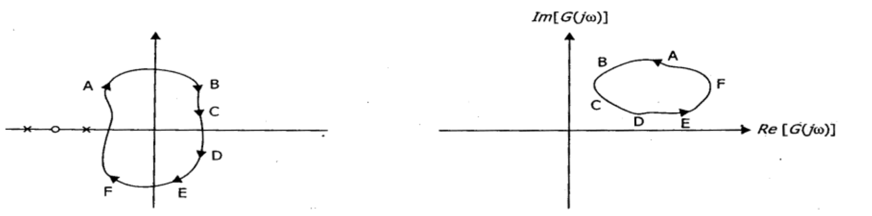
PRINCIPLE OF ARGUMENTS:

It says that if there are ‘p’ poles and ‘z’ zeros are enclosed by the random selected closed path in the S-plane than the corresponding G(s) H(s) lane encircles the origin with (p-z) times.

i.e., N = P-Z

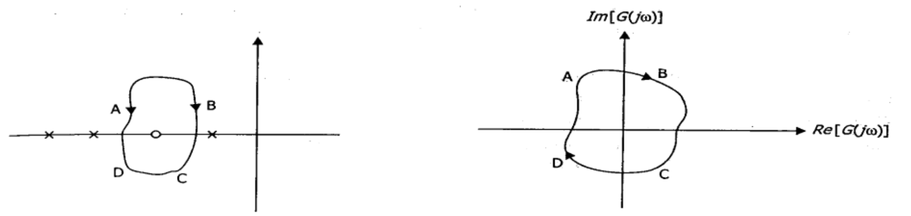
Case 1: No poles and zeros covered

For the right-hand side of the travel direction, the region enclosed by ABCDEF and the corresponding region may be ABCDEF in the G(jω) plane. The origin covered by ABCDEF is important rather than its shape.



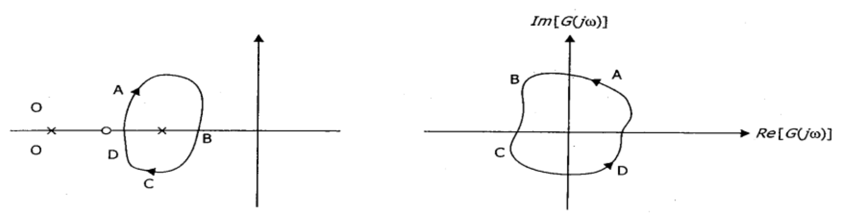
(a) s-plane (b) G(jω) plane

Case 2: One zero only is covered
The origin is enclosed here only once in the clockwise (CW) direction.



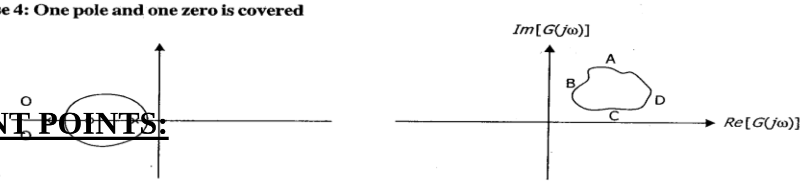
(a) s-plane (b) G(jω) plane

Case 3: One pole only is covered
The origin is enclosed here only once in the counter-clockwise (CCW) direction.



(a) s-plane (b) G(jω) plane

Case 4: One pole and one zero is covered



IMPORTANT POINTS:

- If the GM & PM are very large then the system is more relatively stable but the system response becomes slow.
- If GM & PM is very small then the system is less relatively stable and the system becomes more oscillatory.
- The optimum value of GM & PM is 5 to 10 dB and phase margin is 30^0 to 40^0.

- A system in which all the finite poles and finite zeros lies in the left half of s-plane then it is called **minimum phase** system.
- A system in which one or more poles or zeros lies in the RHS of s-plane then it is called **non-minimum phase** system.
- A system in which all the zeros lies right half of s-plane and all the poles lies in LHS of s-plane which are symmetrical about the imaginary axis then it is called **all pass** system.
- The Bode plot is valid only for minimum phase system
- Non Minimum Phase System = Minimum Phase System X All Pass System
- The frequency at which the magnitude is equal to 1 in linear and 0 is dB is called gain crossover frequency ω_{gc}
- The frequency at which the phase angle is "-180^0" is called the phase cross over frequency ω_{pc}
- Gain margin (GM) = $\left.\frac{1}{|G(j\omega)H(j\omega)|}\right|_{\omega=\omega_{pc}}$

  = 20 log $|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$
- Phase Margin (PM) = 180 + $|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}}$
- If $\omega_{pc} > \omega_{gc} \rightarrow stable \rightarrow GM > 1$ dB and PM is positive
- If $\omega_{pc} = \omega_{gc} \rightarrow marginallystable \rightarrow GM = 1$ dB and PM =0
- If $\omega_{pc} < \omega_{gc} \rightarrow unstable \rightarrow GM < 1$ dB and PM is negative
- The magnitude and phase of phasor G(jω) H(jω) are functions of frequency. As the frequency is varied the tip or edge of phasor traces a locus known as polar plot.
- Using polar plot, it is possible to determine CL stability, relative stability from just OTTF but it is not possible to determine how many roots are causing un-stability. To overcome this difficulty we consider Nyquist plot.
- Addition of pole / zero to the system away from origin rotates the polar plot further by -90^0 / +90^0 at high frequency.
- Addition of pole / zero at origin rotates the polar plot further by -90^0 / +90^0 at all frequencies.
- The number of roots on right side of imaginary axis cannot be determined using polar plot.
- The polar plot will cut the real axis as many times as there are dominant zeros.
- If the arbitrarily chosen closed contour in s-plane encircles 'p' number of poles and 'z' number of zeros of q(s) then the corresponding q(s) contour must encircle the origin (p – z) number of times in anticlockwise direction.
- The total RHS of s-plane as a closed path with radius of '∞' is called "Nyquist contour".
- The Nyquist contour should not pass through either pole or zero.
- To become CL system stable there should not be any closed loop pole in the RHS of s-plane. The CL pole is nothing but zeros of CE which must be zero in the RHS of s-plane, that means z = 0; hence N = P.

PRACTICE QUESTIONS:

1. TF = $\frac{10}{s+2}$, the input is 2Cos (2t + 15^0). Find the steady state output.

2. A system with transfer function $\frac{Y(s)}{X(s)} = \frac{s}{s+p}$ has an output y(t) = $\cos\left(2t - \frac{\pi}{3}\right)$ for the input signal x(t) = p $\cos\left(2t - \frac{\pi}{2}\right)$ Then find the system Parameter 'P'

3. TF = $\frac{10}{s^2(s+1)}$; input is sin(t) find the steady state output

4. The impulse response h (t) of a linear time invariant system is given by h(t) = e^{-2t} u(t), where u(t) denotes the unit step function Find the frequency response H (ω) of the system in terms of angular frequency 'ω'

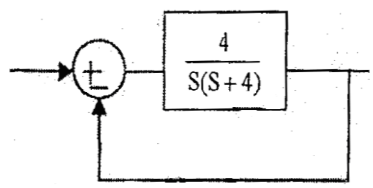
5. CLTF of a certain unity feedback system is $\frac{4}{S^2+7S+13}$. Find the transfer function of the corresponding open loop system with unity feedback.

6. TF = $\frac{s^2+4}{(s+1)(s+4)}$ find the frequency at which the magnitude becomes zero is

7. A system with transfer function G(s) = $\frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$ is excited by sin (ωt). find the frequency at steady – state output of the system is zero

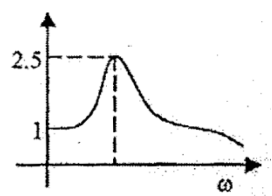
8. CLTF of a certain unity feedback system is $\frac{4}{S^2+7S+13}$. Find the DC gain of the corresponding open loop system

9. Find M_r & ω_r of the system



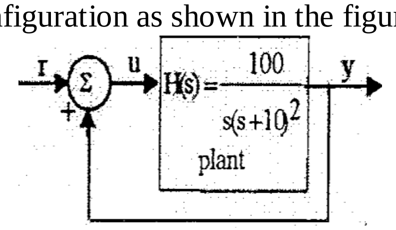
10. G(s) H(s) = $\frac{10}{(s+20)}$ find the Wgc & Wpc

11. TF of a system and its frequency response is given below. Find the value of 'K' and damping ratio $TF = \frac{K\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$



12. G(S) H(S) = $\frac{10}{S(S^2+S+1)}$ find the GM?

13. The input – output transfer function of a plant H(s) = $\frac{100}{s(S+10)^2}$ The plant is placed in a unity Negative feedback Configuration as shown in the figure below



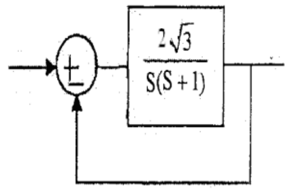
Find the gain margin of the system under closed loop unity negative feedback

14. The open loop transfer function of a unity negative feedback control system is given by

$G(s) = \frac{150}{S(S+9)(S+25)}$. The gain margin of the system is (GATE – IN – 2012)

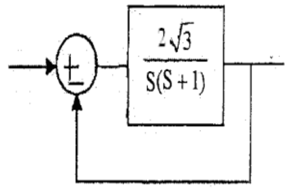
A. 10.8dB B. 22.3dB C. 34.1dB D. 45.6dB

15. Find the gain crossover frequency and PM of the system



16. $G(s)\,H(s) = \frac{10}{(s+20)}$ find the GM & PM

17. Find the phase crossover frequency and GM of the system



18. The GM of the system is 20 dB, if the gain of the system is doubled, then GM becomes

A. 10dB B. 14 dB C. 30dB D. 26 dB

19. $G(S)\,H(S) = \frac{\pi e^{-0.25s}}{S}$ find the GM of the system

20. $G(S)\,H(S) = \frac{10}{S(S+10)}$ find the Phase crossover frequency of the system

21. $G(s)\,H(s) = \frac{\pi e^{-0.25s}}{S}$ Find the GM & PM of the system

22. $G(s)\,H(s) = \frac{10}{S}$ Find the GM & PM of the system

23. $G(S)\,H(S) = \frac{10}{(s+1)(s+20)}$. Find the GM & PM of the system

24. The frequency response of a linear system GH(jω) is provided in the tabular from below

M	Ø
1.3	-130^0
1.2	-140^0
1.0	-150^0
0.8	-160^0
0.5	-180^0
0.3	-200^0

Find the gain margin and phase margin of the system

25. Find the phase margin of a system with the open – loop transfer function

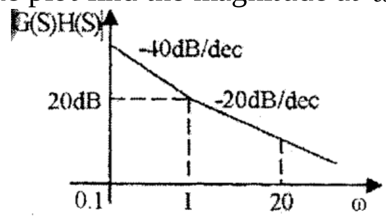
$$G(s)\,H(s) = \frac{(1-s)}{(1+s)(2+s)}$$

26. The open loop transfer function of a unity feedback system is given by

$G(s) = \dfrac{3e^{-2s}}{s(s+2)}$ Find gain and phase crossover frequencies in rad/sec

27. Consider a unity feedback system whose open loop transfer function is $\dfrac{\beta s+1}{s^2}$, find the value of β that results in a phase margin of 45^0.

28. From the following magnitude plot find the magnitude at ω = 0.1rad/sec, and ω = 20 rad /sec



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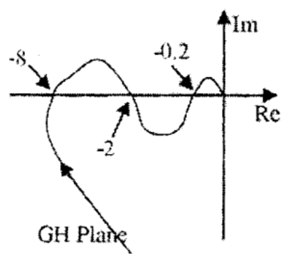
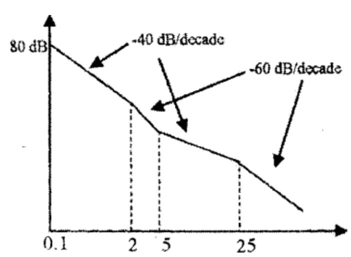
29. The polar diagram of a conditionally stable system for open loop gain K = 1 is shown in figure. The open loop transfer function of the system is known to be stable. Find the range of K such that the closed loop system loop system is stable

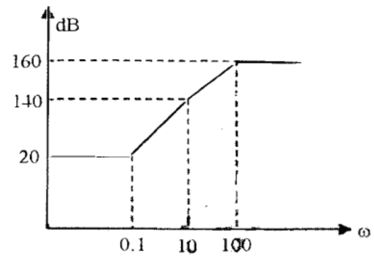


30. The transfer function of a system has the form $G(S) = \dfrac{200(s+2)}{s(s^2+10s+100)}$ at very high frequencies find the slope of Bode gain curve.

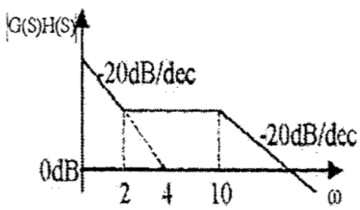
31. The asymptotic approximation of the log magnitude versus frequency plot of a system containing only real poles and zeros is shown, then find its transfer function



32. The approximate Bode magnitude plot of a minimum phase system is shown in figure. Find the transfer function of the system



33. The Bode asymptotic magnitude plot of a critically damped system is shown in figure. Find the system approximated transfer function.



34. Sketch the polar plot of the transfer function G(S) H(S) $= \dfrac{K}{(1+ST)}$; K & T > 0

35. Sketch the polar plot of the transfer function G(S) H(S $= \dfrac{Ke^{-sT_D}}{(S+1)}$

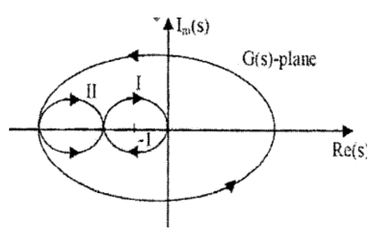
36. Sketch the polar plot of the transfer function G(S) H(S) $= \dfrac{8S}{(S-1)(S-2)}$

37. Find point intersecting of Polar plot of G(S) H(S) = $\dfrac{1}{S(1+s)^2}$ on the negative real axis.

38.. G(S) H(S) = $\dfrac{e^{-sT_D}}{S}$ find the critical value of T_D for the system to be just stable

39. G(S) H(S) = $\dfrac{e^{-sT_D}}{S(S+1)}$ find the critical value of T_D for the system to be just stable

40. The Nyquist plot for the open – loop transfer function G(S) of a unity negative feedback system is shown in figure. If G(S) has no pole in the right half of s – plane , find the number of roots of the system characteristics equation in the right – half of a s – plane



UNT – V

STATE SPACE ANALYSIS

ADVANTAGES :

- It is possible to analyze time-varying (or) time invariant, linear (or) non linear, single (or) multiple input – output system.
- It is possible to confirm the state of the system parameters also and not merely input – output relations.
- It is possible to optimize the systems useful for optimal design.
- It is possible to include initial conditions.
- An n^{th} order differential equation can be converted into n-first order differential equations by using state variable approach. So it can be solved easily using state variables.

DISADVANTAGES:

- Techniques or complex
- Many computations are required.

Basic Concepts:

State:

The state of a dynamic system is the smallest set of variables and the knowledge of these variables at $t = t_0$ together with inputs for $t \geq t_0$ completely determines the behaviour of the system at $r \geq t_0$. A compact and concise representation of the past history of the system can be termed as the state of the system.

State Variables:

The smallest set of variables that determine the state of the system are known as state variables. The knowledge of capacitor, voltage at t = 0, i.e., the initial voltage of the capacitor is a history-dependent term and it forms a state variable. Similarly, initial current in an inductor is treated as a state variable.

State Vector:

The n state variables that completely describe the behaviour of a given system are said to be n components of a vector.

State Space:

The n-dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis, …., x_n axis is known as a state space.

State Model:

$$\overline{X}(t) = AX(t) + BU(t)$$

$$Y(t) = CX(t) + DU(t)$$

$A \rightarrow n \times n$ known as evaluation matrix (or) system matrix

$B \rightarrow n \times m$ known as control matrix (or) input matrix

$C \rightarrow p \times n$ known as output matrix (or) observation matrix

$D \rightarrow p \times m$ known as direct transmission matrix

Different representations of State Model:

- Phase variable in controllable canonical form
- Phase variable in observable controllable form
- Cascade decomposition
- Parallel decomposition
- Jordons canonical form

Transfer Function from State Model:

$$\frac{Y(S)}{U(S)} = C(SI - A)^{-1} B + D$$

Solution of the State Equation:

1. **Homogeneous equation**

   $x(t) = L^{-1} [(SI-A)^{-1}]\, x(0)$ (or)

   $x(t) = \phi(t) \,.\, x(0)$

   Where $\phi(t)$ is state transition matrix $= L^{-1} [(SI-A)^{-1}] = e^{At}$

2. **Non Homogeneous equation**

   $x(t) = L^{-1} [(SI-A)^{-1}]\, x(0) + L^{-1} [(SI-A)^{-1} \,.\, BU(S)]$

   In the above equation on RHS first term represents free response (or) zero input response.
   In the above equation on RHS second term represents forced response (or) zero state response.

   (or)

   $$x(t) = e^{At} x(0) + \int_0^t e^{A(t-x)} Bu(t)\, dt$$

Properties of State Transition Matrix

1. $\phi(0) = 1$
2. $\phi^n(t) = \phi(nt)$
3. $\phi(t_1) \cdot \phi(t_2) = \phi(t_1 + t_2)$
4. $\frac{\phi(t_1)}{\phi(t_2)} = \phi(t_1 - t_2)$

Significance of State Transition Matrix

- State Transition Matrix satisfies the homogeneous state equation, it represents the free response of the system.
- It governs the response that is exited by the initial conditions only.
- State Transition Matrix depends only on the system matrix A.
- State Transition Matrix describes the change of state from the initial time t = 0 to any time t, when the inputs are zero.

Stability from State Variable Approach

The eigen values of the system matrix A are the same as the roots of the characteristic equation which are nothing but the poles of the closed loop transfer function, the stability of a system can be determined by determining the location of the eigen values.

$|SI - A| = 0$ is the characteristic equation of the closed loop system.

Controllability :

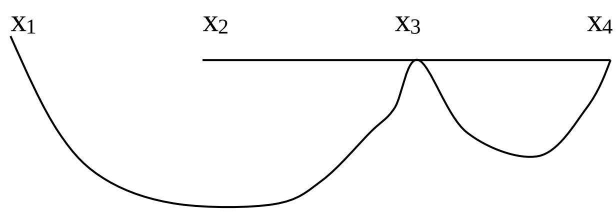
A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $x(t_0)$ to any desired state x(t) in specified finite time by a control vector u(t).
For controllability of the system

$Q_C = [B: AB: \ldots\ldots : A^{n-1} B]$

If the rank of the composite matrix (Q_C) is 'n' then the system is said to be controllable.

Controllability from signal flow graph:

If there is no connection between a certain state and input the system is not controllable.



x_2 is not controllable

Observability:

A system is said to be completely observable, if every state $x(t_0)$ can be completely identified by measurements of the output y(t) over a finite time interval.

For observability of system

$$Q_0 = [C^T : A^T C^T : \ldots\ldots (A^T)^{n-1} C^T]$$

If the rank of the composite matrix (Q_0) is 'n' then the system is said to be observable.

IMPORTANT POINTS:

- The control systems are classified into two types :
  - Open loop control system
    Here, the control action is totally independent of the output.
  - Closed loop control system
    In this case, the controlling action is some how dependent on the output.
- There are two types of feedbacks.
  - Positive Feedback
  - Negative Feedback
- When feedback is given the error between system input and output is reduced. However improvement of error is not only advantage. The effects of feedback are
  - Gain is reduced by a factor $\frac{G}{1 \pm GH}$
  - Reduction of parameter variation by a factor $1 \pm GH$.
  - Improvement in sensitivity.
  - Stability may be affected.
- **Test of controllability**
  A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $x(t_0)$ to any other desired state $x(t_1)$ in specified time by a control vector (say u(t)).
  Test for Controllability
  For an nth order system to be controllable, rank of the matrix [B: AB,….. A^{n-1} B] should be n.
- **Test of Observability**
  A system is said to be completely observable, if every state $x(t_0)$ can be completely identified by measurements of the output y(t) over a finite time interval.
  Test of observability
  For 2nda order system, Rank of $\begin{vmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{vmatrix}_{npxn}$ be n.
- **Test for Stability**
  For stability analysis, solve |SI – A| = 0 and get the characteristic equation.
  Using Routh's array, we can find the stability of the system.

PRACTICE QUESTIONS:

1. Obtain the SSR of a system described by the following differential equation $\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y$ = u(t) by CCF,OCF and DCF Normal forms.

2. SSR of a certain system is described by $\frac{dx(t)}{dt}$ = -3X(t)+0.5 u(t),y(t)=-2x(t). Then find the TF.

3. The TF&SSR of a system is $\frac{Y(t)}{U(s)} = \frac{1}{s^2+3s+2}$,X(t) = $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$U(t), Y(t) = $[C_1 C_2]$X(t).Then find the elements C_1&C_2 .

4. Obtain a state space representation in diagonal form for the following system $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = 6u(t)$.

5. The SSR &TF of a system are $X(t) = \begin{bmatrix} 0 & 1 \\ -4 & -a \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$U(t),Y(t) = [2 2] X(t) , TF = $\frac{2s+2}{S^2+5S+4}$. Then find the value of a.

6. ϕ (t) = e^{At}, find ϕ (t) and ϕ^{-1} (t) if ϕ^2 (t) = $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-4t} \end{bmatrix}$.

7. The state equation is X (t) = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$X(t), Find the zero input solution of the state equation ,if X(0) = $[0\ 0]^T$.

8. The state equation is X (t) = $\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$X(t) + $\begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}$U(t). Find the state transition matrix.

9. State equation is X(t) = $\begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$ X(t) + $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$U(t) & X(0) = $[-1\ 3]^T$. Find the solution to a step input.

10. ***The TF of a certain 2nd order system is*** $\frac{S+1}{S^2+4S+5}$ ***. Find the property of controllability and observability.***

11. ***SSR is X (t) =*** $\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$ ***X(t) +*** $\begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}$***U(t),Y(t) =*** $\frac{10}{s+2}$ ***X(t). Find the property of controllability and observability.***

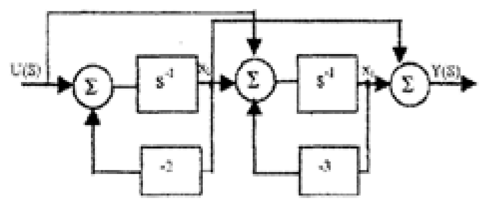
12. ***SSR of a system is***

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} U(t), Y(t) = [1 \;\; 0 \;\; 2] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

. Find the property of controllability and observability.

13. ***SSR of a system is*** $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \begin{bmatrix} y1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$. ***Find the property of controllability and observability.***

14. ***Obtain state model of the state diagram of a system is given below:***



15. ***Given the homogeneous state-space equation X*** $= \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$***x . Find the steady state value of*** $X_{ss} = \lim_{t \to \infty} x(t)$ **,** given the initial state value of x(0) = $[10 \; -10]^T$

16. A second order system starts with an initial condition of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ without any external input.The state transition matrix for the system is given by $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}$. Find the state of the system at the end of 1 second.

17. ***A system is decribed by the following state and output equations.***

$\frac{dx_1(t)}{dt}$ ***= -3x₁(t)+x₂(t)+2u(t)***

$\frac{dx_2(t)}{dt}$ ***= -2 x₂(t)+u(t)***

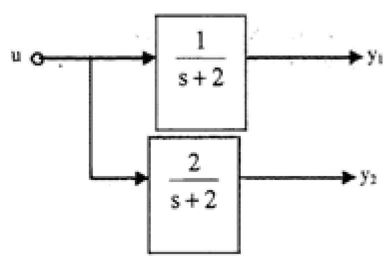
y(t) = x₁(t) when u(t) is the input and y(t) is the output. Find the system transfer functionand the state-transtion matrix of the system

18. ***A linear time-invariant system is described by the state variable model*** $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ***u,Y = [1 2]*** $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$. ***Find the property of controllability and observability***

19. ***Find the zero-input response of a system given by the state space equation*** $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ ***and*** $\begin{bmatrix} X_1 & (0) \\ X_2 & (0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

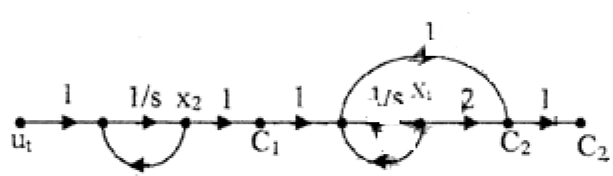
20. ***A linear system is described by the following state equation X(t) = AX(t)+BU(t),*** ***A =*** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$***.Find the state transition matrix of the system.***

21. ***The block diagram of a system with one input u and two outputs*** y_1 ***and*** y_2 ***is given below.***



Obtain the state space model of the above system in terms of the state vector x and the output vector $\underline{y} = [y_1\ y_2]^T$ ***.***

22. ***The state diagram of a system is shown in the given figure :***



Find the property of controllability and observability.

.

23. ***Consider the single input single output system with its state variable representation:***

$x = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} U$, ***Y = [1 0 2] X. Find the property of controllability and observability.***

24. ***Obtain the state-space representation in phase variable from the transfer function G(s)*** $= \dfrac{2s+1}{s^2+7s+9}$

25. ***The state equations of a system are given by*** $x = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$, ***y = [1 0 1]x. Find the property of controllability and observability.***

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26. ***Given*** $[x] = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} [x] + \begin{bmatrix} 0 \\ k \end{bmatrix} u$, $y = x_1 + x_2$, $[X] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. ***What is the transfer y /u?***

27. ***The state-variable description of a linear antonomous system is X = AX where X is two-dimensional state vector and A is a matrix given by*** $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$. ***Find the pole of the system.***

28. ***A linear system is described by the following state equations .X (t)*** $= \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} X + \begin{bmatrix} 2 \\ 0 \end{bmatrix} U$, ***Y(t)= [0 3]X. Then find the TF Y(s)/U(s)***

29. ***The system matrix of a continuous time system is given by*** $A = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}$. ***Then write the characteristic equation and find the stability of the system.***

30. ***Obtain the diagonal form of the state space representation of a SISO system with the transfer function*** $\dfrac{Y(s)}{U(s)} = \dfrac{3s^2+6s+2}{s^3+3s^2+2s}$

31. ***Find the transfer function of the system with the state space representation***

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u;$$

y = [1 1]x+u

32. ***The state variable description of an LTI system is given by*** $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ***u ; y = (1 0 0)*** $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ ***where y is the output and u is the input. Find condition such that the system is controllable***