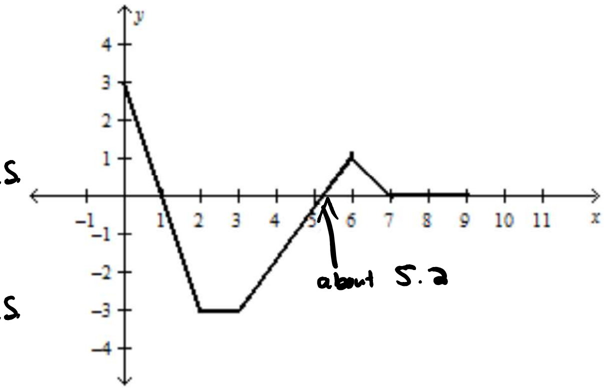
Name____________________________________Date____________________Class___________

Homework 5.5

The function whose graph is pictured below, represents the velocity, $v(t)$, of a particle for $t = 0$ to $t = 9$ seconds moving along the x – axis. Use the graph to complete exercises 1 – 4.



1. On what interval(s) is the particle moving to the right? Left? Justify your answer.

• $v(t) > 0$ on $(0,1) \cup (5.2, 7)$

∴ The particle is moving right on these intervals

• $v(t) < 0$ on $(1, 5.2)$

∴ The particle is moving left on this intervals

2. On what interval(s) is the particle slowing down? Speeding up? Justify your answer.

$v(t) > 0$ when $v(t)$ is above t-axis. $v(t) < 0$ when $v(t)$ is below t-axis

$a(t) > 0$ when $v(t)$ is increasing. $a(t) < 0$ when $v(t)$ is decreasing.

• The particle is slowing down when $v(t)$ and $a(t)$ are opposite signs.

$v(t) > 0$ and $a(t) < 0$ on $(0,1) \cup (6,7)$

$v(t) < 0$ and $a(t) > 0$ on $(3, 5.2)$

∴ The particle is slowing down on $(0,1), (3, 5.2) \cup (6,7)$

• The particle is speeding up when $v(t)$ and $a(t)$ are the same signs.

$v(t) > 0$ and $a(t) > 0$ on $(5.2, 6)$

$v(t) < 0$ and $a(t) < 0$ on $(1,2)$

∴ The particle is speeding up on $(1,2)$ and $(5.2, 6)$

3. At what value(s) of t is the particle momentarily stopped and changing directions? Justify your answer.

$v(t)$ crosses the t-axis at $t = 1$ and $t = 5.2$

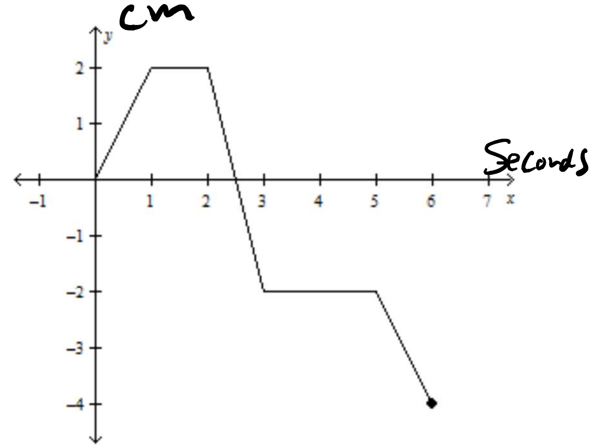
∴ The particle momentarily stops and changes direction at $t = 1$ and $t = 5.2$

4. On what interval of the time is the acceleration 0? Justify your answer.

$v(t)$ is constant on $(7, 9)$

∴ $a(t) = 0$ on $(7, 9)$

The graph below represents the position, $p(t)$, of a particle that is moving along the x – axis on the interval $0 \le t \le 6$. Use the graph to complete exercises 5 – 9. $p(t)$ is measured in centimeters and t is measured in seconds.



5. For what interval(s) of time is the particle moving to the right? Justify your answer.

$v(t) > 0$ on $(0,1)$

$\therefore$ particle is moving right on $(0,1)$

6. For what interval(s) of time is the particle moving to the left? Justify your answer.

$v(t) < 0$ on $(2,3) \cup (5,6)$

$\therefore$ particle is moving left on $(2,3) \cup (5,6)$

7. Express the velocity, $v(t)$, as a piecewise-defined function on the interval $0 < t < 6$.

$$v(t) = \begin{cases} 2, & 0<t<1 \\ 0, & 1<t<2 \\ -4, & 2<t<3 \\ 2, & 3<t<5 \\ -2, & 5<t<6 \end{cases}$$

8. At what value(s) of t is the velocity undefined on the interval $1 < t < 6$? Graphically justify your reasoning.

$p(t)$ has cusps at $t=1$, $t=3$, and $t=5$

$\therefore$ $v(t)$ is undefined at $t=1$, $t=3$, and $t=5$

9. Find the average velocity of the particle on the interval $1 \le t \le 6$.

Average Velocity on $[1,6] = \dfrac{p(1)-p(6)}{1-6} = \dfrac{2-(-4)}{-5} = \dfrac{6}{-5}$ cm/sec

A particle moves along the x – axis so that at any time $0 \le t \le 5$, the velocity, in meters per second, is given by the function $v(t) = (t-2)^2 \cos(2t)$. Use a graphing calculator to complete exercises 10 – 12.

$2(t-2)^1(1)\cos(2t) + (t-2)^2(-\sin(2t)\cdot 2)$

10. On the interval $0 \le t \le 5$, at how many times does the particle change directions? Give a reason for your answer.

v(t) crosses the t-axis at $x = \frac{\pi}{4}$, $x = 2$, $x = \frac{3\pi}{4}$, and $x = \frac{5\pi}{4}$

$\therefore$ The particle changes direction 4 times on $[0, 5]$

11. Using appropriate units, what is the value of $v'(2)$. Describe the motion of the particle at this time. Justify your answer.

$v'(2) = 0$ meters/second^2

$v'(2) = 0$ and $v(2) = 0$

$\therefore$ The particle is not moving at all.

12. Using appropriate units, what is the average acceleration between $t = 1$ and $t = 3.5$ seconds?

Average Acceleration on $(1, 3.5) = \dfrac{v(1) - v(3.5)}{1 - 3.5} = \dfrac{-.416 - 1.696}{-2.5} = \dfrac{-2.112}{-2.5}$

≈ 0.845 meters/second^2

13. What is the acceleration of the particle the first time that the velocity is 0?

The first time $v(t) = 0$ is when $t = \pi/4$

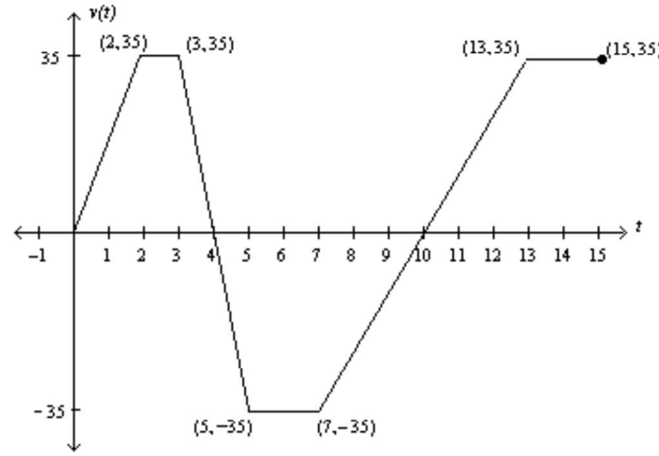
$a(t) = 2(t-2)^1(1)\cos(2t) + (t-2)^2(-\sin(2t)\cdot 2)$

$a(t) = 2(t-2)\cos(2t) - 2(t-2)^2\sin(2t)$

$a(\frac{\pi}{4}) = 2(\frac{\pi}{4} - 2)\cos(2\cdot\frac{\pi}{4}) - 2(\frac{\pi}{4} - 2)^2\sin(2\cdot\frac{\pi}{4})$

$a(\frac{\pi}{4}) \approx -2.951$

Jeff leaves his house riding his bicycle toward school. His velocity $v(t)$, measured in feet per minute, on the interval $0 \le t \le 15$, for t minutes, is shown in the graph to the right. Use the graph to complete exercises 14 – 17.



14. Find the value of $v'(4)$. Explain, using appropriate units, what this value represents.

$v'(4) = \frac{35-(-35)}{3-5} = \frac{70}{-2} = -35$ feet/min^2

Four minutes after leaving home, Jeff's acceleration is -35 feet/min^2.

15. On the interval $0 \le t \le 5$, is there any interval of time at which $a(t) = 0$? Explain how you know.

$v(t)$ is constant on $(2,3)$

$\therefore$ $a(t) = 0$ on $(2,3)$

16. On the interval $0 \le t \le 5$, does Rolle's Theorem guarantee that there will be a value of t such that $a(t) = 0$? Justify your answer.

1. $v(t)$ is not differential for all values of t on $(0,5)$
2. $v(0) \neq v(5)$

$\therefore$ Rolle's Theorem does NOT guarantee a value of t such that $a(t)=0$

17. At some point, Jeff realizes that he forgot something at home and has to turn around. After how many minutes does he turn around? Give a reason for your answer.

$v(t) > 0$ on $(0,4)$ which means Jeff's position from home is increasing.

$v(t) < 0$ on $(4,5)$ which means Jeff's position from home is decreasing.

$\therefore$ Jeff's velocity changes signs at $t=4$.

$\therefore$ Jeff turns around at $t=4$.